Money with Flexible prices

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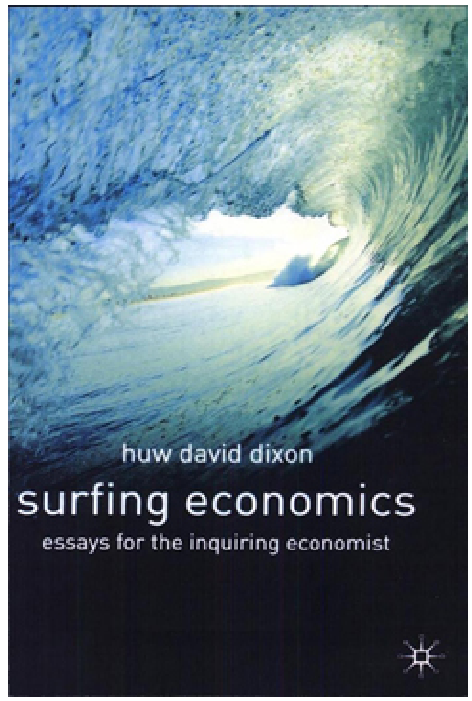
1 Intro: a quick history of everything.

- Full details: Dixon *surfing economics* on huwdixon.org: chapters 3 and 4.



- Pre-Keynes: *Classical Dichotomy* (David Hume and before). Money Neutral: real things are determined by real things, money just affects prices.

- Perfect competition Prices and wages flexible, competitive equilibrium ("markets clear through price mechanism"

- 1936 Keynes: invents macroeconomics (national income accounts etc.).

- Nominal rigidity. Markets do not clear, so get excess supply (involuntary unemployment).

- Over subsequent years: development of the neoclassical synthesis (Hicks, Don Patinkin, Hansen...40s and 50s).

- Short run: prices and or wages fixed (IS/LM, AD/AS)
- Long-run: all prices clear, get to competitive equilibrium (vertical long-run AS curve).

- 1970s: rational expectations (*new classical*): kept the microeconomic structure of neoclassical synthesis, but introduced RE. With RE, even in IS/LM or AD/AS type of model, "systematic monetary policy" is neutral, only unsystematic (unpredictable)policy affects output. Barro: Ricardian equivalence.

- 1980s Real Business Cycle theory. Forget about money alltogether. Developed explicitly dynamic model of the representative household (Euler equations etc.).

- 1980s. New Keynesian macro: introduced *imperfect competition*. Need this to explore micro-foundation of price rigidity. No longer assume perfect competition...taylor rules, Calvo, menu costs etc.

- 1990s: new neoclassical synthesis: combine RBC with new Keynesian models. Long-run steady-state: money neutral. Short-run: wage and price dynamics determined by the new Keynesian dynamic pricing models.

- This lecture: look at basic dynamic model with flexible prices.

2 Money

- Why does money exist? Two answers from a modelling perspective.

- Money gives utility (MIU: Money in Utility). This dates back to the 1950s Don Patinkin (Money Interest and Prices) in static models, and Sidrauski in dynamic models.

- Money is part of a transactions technology (CIA: Cash in advance).

- We will first look at models with flexible market clearing prices.

3 MIU Model.

- We include two nominal assets:

- money (which pays no interest but yields utility) and Bonds (which pay interest but have no utility).

- Lifetime utility (leave out leisure)

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}\right) \tag{1}$$

- Budget constraint.

$$Y_t + (1-\delta)\, K_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t}$$

Note: there is a distinctin between "beginning of period" and "end of period". The LHS is the total available at the beginning of period t, the RHS what it ends up in at the end of period t. Note, it is the end-of period money balances that matter for utility (not obvious why!).

- Production function: With a fixed labour supply normalized to unity

$$Y_t = F\left(K_{t-1}, 1\right) = f(k_{t-1})$$

output in period t depends on the end of period capital from period $t-1$ (the start of period capital for period t).

- We can write the budget constraint as:

$$c_t + k_t + m_t + b_t - \tau_t = f\left(k_{t-1}\right) + \left(1 - \delta\right) k_{t-1} + (1+i)\frac{b_{t-1}}{1+\pi_t} + \frac{m_{t-1}}{1+\pi_t} \tag{2}$$

where $b_{t-1} = B_{t-1}/P_{t-1}$ and $m_{t-1} = M_{t-1}/P_{t-1}$, $\pi_t = \left(P_t - P_{t-1}\right)/P_{t-}$ hence

$$\frac{m_{t-1}}{1+\pi_t} = \frac{M_{t-1}}{P_t}$$

For convenience define

$$\omega_t = f\left(k_{t-1}\right) + \left(1-\delta\right)k_{t-1} + (1+i)\frac{b_{t-1}}{1+\pi_t} + \frac{m_{t-1}}{1+\pi_t}$$

- Set up inter-temporal maximization: subsitute out k_t

$$k_t = \omega_t - c_t - m_t - b_t$$

get first order conditions (set $\delta = 1$)

$$H_c = \beta^t U_{Ct} - \lambda_t = 0 \tag{3a}$$

$$H_m = \beta^t U_{mt} - \lambda_t + \lambda_{\ t+1}\frac{1}{1+\pi_{t+1}} = 0 \tag{3b}$$

$$H_b = \lambda_{t+1}\frac{1+i_t}{1+\pi_{t+1}} - \lambda_t = 0 \tag{3c}$$

$$H_k = -\lambda_t + \lambda_{t+1} f_k\left(k_t\right) = 0 \tag{3d}$$

Transversality

$$\underset{t \to \infty}{Lim} \beta^t \lambda_t k_t = 0$$

Likewise for m_t and b_t.

- Easy interpretation:

$$\frac{1 + i_t}{1 + \pi_{t+1}} = f_K(k_t) \tag{4}$$

arbitrage: $(3d, 3c)$ equates the rate of return on the two assets - bonds and capital.

- From $(3a, 3b)$ you get

$$U_{Ct} = U_{mt} + \beta \frac{U_{Ct+1}}{(1 + \pi_{t+1})} \tag{5}$$

The RHS represents the overall utility from more money now: you get the direct utility from money now, plus the extra money to start next period (you discount it, inflation may erode it, but you get extra consumption). This is equated to the marginal utility of consuming now.

- we can rewrite (5) as:

$$1 = \frac{U_{mt}}{U_{Ct}} - \frac{1}{(1+\pi_{t+1})} \frac{\beta U_{Ct+1}}{U_{CT}}$$

From ($3d$) this gives us

$$1 = \frac{U_{mt}}{U_{Ct}} - \frac{1}{(1+\pi_{t+1})} \frac{1}{f_{Kt}}$$

Hence from the arbitrage condition (4) we get

$$\frac{U_{mt}}{U_{Ct}} = \frac{i}{(1+i)} \tag{6}$$

This means MRS between consumption and real balances equals the relative price (opportunity cost) of money and consumption.

- The house hold could hold one more unit of money, which means that it holds less bonds or capital (which both have the same rate of return) $f_K(1+\pi_{t+1})$. If it holds money, it receives no interest (it gets 1 for 1). Hence the opportunity cost of money is the reciprocal of the nominal rate of return on bonds/capital.

- Euler condition:

$$\frac{U_{Ct}}{\beta U_{Ct+1}} = f_{Kt}$$

- Arbitrage: rate of return on bonds equals that on capital

$$\frac{1+i}{1+\pi} = f_K$$

Hence if we define $r_t = f_{Kt} - 1$ (the rate of return on capital)

$$\begin{aligned} (1+i) &= (1+r_t)(1+\pi_t) \\ i &\approx r_t + \pi_t \text{ (if small)} \end{aligned}$$

Fisher equation (1896): the nominal rate of interest equals the real rate plus inflation.

- In aggregate, bonds equal zero (closed economy: all owned by people in the economy: money is the only net financial asset): in aggregate

$$c_t + k_t + m_t = f(k_{t-1}) - \tau_t + \frac{m_{t-1}}{1+\pi_t} \tag{7}$$

4 Steady State in MIU.

Drop time subscripts

$$
\begin{aligned}
f_K &= \frac{1}{\beta} \\
\frac{1+i}{1+\pi} &= \frac{1}{\beta} \\
\frac{U_{mt}}{U_{Ct}} &= \frac{\beta}{(1+\pi)}
\end{aligned}
\tag{8}
$$

aggregate budget constraint, along with assumption that steady state money growth equals inflation (real money balances are constant):

$$
c + k + m = f(k) - \tau + \frac{m}{1+\pi}
$$

- government budget constraint

$$\tau = m - \frac{m}{1+\pi}$$

$$c + k = f(k)$$

- Real and monetary seperate in steady state: government budget cosntraint means that taxes and real money creation exactly offset each other, so that output in aggregate divides between consumption and steady state investment.

- Classical Dichotomy: all real variables determined by real things. Model is neutral (level of money does not affect real variables) and is superneutral (rate of growth does not affect real variables).

- Real money balances: for given steady state money growth, is there a level or real money balances that satisfies (8)? Clearly, π will influence SS MRS between money and consumption.
  - A Sufficient condition for existence for $\pi \geq 0$ is separable utility $(U_{cm} = 0)$ and $U_m(C, 0) = \infty$ and

$$\underset{m \to \infty}{Lim} U_m < 0$$

  - Log utility: $U(C, m) = u(C) + \log m$. Always a solution despite $U_m > 0$.

- Is there more than one solution? Yes, there can be!

- Money demand: nominal interest rate and consumption

$$U_{mt} = U_{Ct}\frac{i}{1+i}$$

If U is CES

$$U = \left[aC_t^{1-b} + (1-a)\,m_t^{1-b}\right]^{\frac{1}{1-b}}$$

The MRS is

$$\frac{U_m}{U_C} = \left(\frac{1-a}{a}\right)\left(\frac{c_t}{m_t}\right)^b$$

yielding money demand

$$m_t = \left(\frac{1-a}{a}\right)^{\frac{1}{b}}\left(\frac{i}{1+i}\right)^{-\frac{1}{b}}C_t \tag{9}$$

so that the interest elasticity of demand is b^{-1}. Estimated by Kari Kehoe and McGrattan (2000) estimate the interest elasticity of demand to be about 0.39.

4.1 Endogenous labour

- Output and productivity

$$
\begin{aligned}
Y_t &= e^{zt} F\left(K_t, L_t\right) \\
z_t &= \rho z_{t-1} + e_t
\end{aligned}
$$

- Utility

$$
U\left(C_t, m_t, \ell\right) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + b\frac{m_t^{1-\Phi}}{1-\Phi} - \Psi \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}
$$

- Money Supply:

$$
\begin{aligned}
m_t + \pi_t &= u_t \\
u_t &= \gamma u_{t-1} + \phi_t
\end{aligned}
$$

$m_t + \pi_t$ is deviation of the nominal money supply from ss. Hence

5 Dynamics.

- Production Function.

$$y_t = \alpha k_{t-1} + (1 - \alpha) n_t + z_t$$

- Accumulation equation

$$\begin{aligned} k_t &= \frac{Y}{K} y_t + \frac{C}{K} c_t \\ &= \alpha \frac{Y}{K} k_{t-1} + (1-\alpha) \frac{Y}{K} n_t + \frac{C}{K} c_t + \frac{Y}{K} z_t \end{aligned}$$

- $\Lambda_t = U_{ct}$

$$\lambda_t = \frac{1}{\sigma} c_t$$

- MRS (C, m)

$$\begin{aligned} \frac{U_{mt}}{U_{Ct}} &= \frac{1}{(1+\pi_{t+1})} \frac{\beta E U_{Ct+1}}{U_{Ct}} \\ E\pi_{t+1} &= \Phi m_t + \sigma^{-1} E_t c_{t+1} \end{aligned}$$

- MRS (C, L)

$$\begin{aligned} \frac{U_{\ell t}}{U_{Ct}} &= F_L \\ \eta n_t + \sigma^{-1} c_t &= \phi_{Lk} k_{t-1} + \phi_{LL} n_t \end{aligned} \tag{10}$$

NB: Can express F_L either as a function of K or Y. In C-D format, $F_L = (1-\alpha) Y/N$.

- Euler

$$\frac{1}{\sigma} (E_t c_{t+1} - c_t) = \omega (E_t n_{t+1} - k_t)$$

where ω is the elasticity of the MPK with respect to (K/L). For C-D, we have

$$\begin{aligned} F_K &= \frac{\alpha}{(1-\alpha)} \frac{L}{K} \\ \omega &= 1 \end{aligned}$$

- Fisher

$$i_t = \omega \left(E_t n_{t+1} - k_t \right) + E_t \pi_{t+1}$$

- The model has one endogenous state variable (k), one exogenous state variable (m evolves according to the autonomous money growth process). endogenous jump variables are (n, c, π) . Exogenous variables are the shocks (u, ϕ).

- Can eliminate n_t using (10).

5.1 Solution.

$$
\begin{aligned}
k_t - \alpha \frac{Y}{K} k_{t-1} - (1-\alpha) \frac{Y}{K} n_t - \frac{C}{K} c_t - \frac{Y}{K} z_t &= 0 \\
k_t - E_t n_{t+1} - \frac{1}{\sigma} (E_t c_{t+1} - c_t) &= 0 \\
-\phi_{Lk} k_{t-1} + [\eta - \phi_{LL}] n_t + \sigma^{-1} c_t &= 0 \\
z_t - \rho z_{t-1} - e_t &= 0 \\
E \pi_{t+1} - \Phi m_t - \sigma E_t c_{t+1} &= 0 \\
m_t + \pi_t - u_t &= \\
u_t - \gamma u_{t-1} - \phi_t &= 0
\end{aligned}
$$

Note: this model decomposes: can solve the real part (k, c): then given evolution of m can solve for π. Equations 1-4 are just like an RBC model and can be solved accordingly. Equations $5, 6$ give evolution of m and π when path of consumption and money are known. Prices are perfectly flexible: money is neutral. Money has no effect on the real side of the economy.

- In Walsh's version of Sidrauski model, money is not neutral in this way. That is because $U_{mc} \neq 0$, so that a shock to the money supply causes the marginal utility of consumption to change. A monetary shock causes inflation expectations to rise (if there is serial correlation $\gamma > 0$, otherwise there is just a one off effect to the price level). This leads to a fall in real balances, which reduces the marginal utility of consumption: the house hold consumes less and works less (more leisure), so that output falls.

- The real interest rate falls (lower labour supply), the Fisher equation implies that inflation rises by more than the nominal intersst rate.

6 Conclusion.

- You can include money in an RBC framework using MIU. However, with seperability of utility function get complete neutrality of money. If money and cosnumption are not seperable (as in Walsh's example) can get non-neutrality in response to a money growth shock; but very small effect.

- Need to consider dynamic wage-setting and pricing models. The underlying model is usually MIU with seperable preferences. Hence with perfectly flexible prices money is neutral both in steady-state and dynamics.