

1 The Multiplier in an Economy with Monopolistic Output Markets and Competitive Labour Markets

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1.1 Introduction

The purpose of this paper is to derive some new results and to link together these results with an existing literature. The focus of the paper is a macro-economy characterized by monopolistic competition in output markets, but with a perfectly competitive labour market. We explore the implications of imperfect competition for the conduct and effectiveness of fiscal policy for output and employment in both the short run (with the number of firms fixed) and the long run (with free entry and exit of firms).¹

This paper develops the analysis of Dixon and Lawler (1996): the extension is primarily in the introduction of a monetary sector into the model (Dixon and Lawler, 1996, has only output and leisure). The origins of this analysis lie in three papers: Startz (1989), Dixon (1987) and Mankiw (1988). In the papers by Dixon and Mankiw, the relationship between the fiscal multiplier and the degree of competition, was analyzed in the short run with a fixed number of firms. Startz (1989) examined the effect of free-entry on the model. In fact, all three papers share certain key assumptions which are crucial for the results which they obtain, as shown by Dixon and Lawler (1996). In particular they assume (a) constant marginal product of labour (CMPL) production technology and (b) household preferences which give rise to constant marginal budget shares.² We denote these two assumptions as defining the SMD (Startz - Mankiw - Dixon) approach. Two key conclusions emerged from the SMD analysis:

1. the short-run fiscal multiplier is larger the greater the degree of monopoly - Dixon (1987, p. 144, Proposition 3), Mankiw (1988, pp. 10-11, Equations (15), (16) and (17)), Startz (1989, p. 744, Equation (11)).

2. in the presence of monopoly power the short-run fiscal multiplier exceeds the corresponding long-run multiplier (Startz, pp. 749-50).

Dixon and Lawler (1996) examined the generality of these results using a model which encompassed the SMD framework as a special or limiting case. One of the main innovative features of that model lay in its treatment of firms' production technology. As in Startz a fixed production cost was assumed, but this was combined with the assumption of diminishing marginal productivity of labour. Together these assumptions imply the familiar U-shaped average cost curve and upward sloping marginal cost curve. Given this specification of technology, the long-run free entry condition serves to determine, independently of the rest of the model, the long-run equilibrium values of the real wage, and employment and output per firm. An appealing feature of these technological assumptions is that they are perfectly consistent with both monopolistic and perfect competition in contrast to Startz's framework, which is incompatible with a Walrasian equilibrium.

In this paper we make the same assumptions about technology and preferences as in Dixon and Lawler, but we employ our framework to examine the effects of fiscal policy in both the short and long runs. In so doing we show that neither of the conclusions (1) and (2) referred to above are general propositions about the impact of fiscal policy in an imperfectly competitive economy; rather both results reflect the particular assumptions with regards to household preferences and production technology which characterize the SMD framework. In particular we demonstrate, first, that there is no unambiguous relationship between the size of the fiscal multiplier (either short or long run) and the degree of monopoly power. Secondly, we show that Startz's ranking of the short and long-run output multipliers is reversed for 'sufficiently competitive' monopolistic economies. Thirdly, we find an unambiguous ranking of employment multipliers: in particular the long-run employment multiplier is always, that is regardless of the degree of monopoly power, larger than the corresponding short-run multiplier. Finally, we indicate the relationship between our own conclusions and the SMD results by demonstrating how each of the latter derives from the particular assumption of CMPL technology, constant marginal expenditure shares, or both. This confirms most of the results of Dixon and Lawler (1996) within the context of a monetary economy.

In Section 1.5 of the paper, we present a geometric and visual representation and derivation of the results of the paper. Whilst the main body of the paper uses a very general representation of preferences, in the graphical analysis we adopt the special case of homothetic preferences. We show how the relationship between the multiplier and the degree of imperfect competi-

tion depends crucially on division of full income between consumption and leisure by the household. In Section 1.6, we relate the static results to the continuous time intertemporal dynamic framework. This is based very much on Dixon (1997), and relates to the discrete-time framework of Rotemberg and Woodford (1995) and Devereux et al. (1996). We develop a formal and geometric analysis of the steady-state long-run multipliers, and show that the analysis is very similar to the static case, except for new factors introduced by capital accumulation and intertemporal preferences. However, we note that much of the literature in the intertemporal macroeconomic setup often makes strong assumptions about the functional form in order to solve or simulate the model (these often include Cobb-Douglas intra-temporal preferences and technology). The static analysis of Dixon and Lawler (1996) and this paper would indicate that the results of these papers might be very limited by the specific nature of these assumptions.

1.2 The model

In this section we outline the central assumptions which characterize our model of a monopolistically competitive economy: the Walrasian case of perfect competition can be viewed as a special limiting case within the framework, and will be treated as such. Briefly, three sets of agents, households, firms and the government interact in the markets for labour, goods and money. Whilst the labour market is taken to be perfectly competitive, the goods market is assumed to be populated by Dixit-Stiglitz monopolistic competitors, whose output is purchased by both households and government. Households, who act as price takers, consume goods, sell labour and hold money, the only asset in the model. Money is issued by the government and provides an alternative to lump-sum taxation as a means of financing government expenditure. We now turn to consider the individual components of the model in some detail.

1.2.1 Households

Our model of product-differentiation is essentially that of Judd (1985) which develops the basic Dixit-Stiglitz model (see Grossman and Helpman, 1991 for a useful exposition). This model is then embedded within a more-or-less standard treatment of the 'macroeconomic consumer.' There is a continuum of firms, indexed by j , uniformly distributed over the interval $[0, n]$, where n is the measure ('number') of firms. Each firm j sets price $p(j)$ and produces output $y(j)$. All households are taken to be identical and hence

their behaviour can be encapsulated in the form of a single representative household, whose preferences over consumption goods are assumed to be separable from the other arguments of the utility function. We define the sub-utility function C :

$$C = n^{\frac{\mu}{\mu-1}} \left[\int_0^n c(j)^{1-\mu} dj \right]^{\frac{1}{1-\mu}} \quad (1.1)$$

where $c(j)$ represents household consumption of firm j 's output and μ , assumed to lie in the half-open interval $[0, 1)$, is a preference parameter. For $\mu = 0$, firms outputs are viewed as perfect substitutes (hence C provides a direct measure of total consumption) and as μ increases the degree of substitutability between different goods declines.

Household utility is a function of the consumption index C , defined by (1.1), leisure (that is the household time-endowment, E , less labour supply L^s) and real money balances, that is nominal end-of-period money holdings, M , deflated by an appropriate price index P , to be defined below.

A1 : Household Preferences:

$$U : \mathbb{R}_+^3 \rightarrow \mathbb{R}_+ \text{ where } U = U \left(C, E - L^s, \frac{M}{P} \right)$$

with U (at least) twice continuously differentiable, strictly quasi-concave and increasing in each of its arguments.

In subsequent sections a special case of A1 is used on occasion as a reference point, namely that U is Cobb-Douglas

$$U = C^\alpha (E - L^s)^\beta \left(\frac{M}{P} \right)^\gamma, \quad \alpha, \beta, \gamma > 0, \alpha + \beta + \gamma = 1$$

As already noted, Cobb-Douglas preferences are assumed in Dixon and Man-kiw, with Startz's Stone-Geary specification equivalent for all relevant purposes.³ Other New Keynesian papers also use special cases of A1. Blanchard and Kiyotaki, for example, assume preferences to be additively separable, with utility Cobb-Douglas with respect to consumption and real money holdings but linear in leisure.⁴

The households budget constraint is

$$\int_0^n p(j) \cdot c(j) dj + M \leq W \cdot L^s + \Pi + M^0 - T \quad (1.2)$$

where W is the nominal wage rate, Π the nominal value of distributed profits, M^0 initial holdings of nominal money balances and T the nominal value of

lump-sum taxation. The household maximizes utility as described by A1, subject to (1.1) and (1.2).

Since preferences are separable the households decision process can be represented as a two-stage budgeting problem.⁵ In the first stage the household chooses, given its endowments, the optimal values for consumption expenditure, leisure and money balances. Then, in the second stage, it allocates its consumption expenditure between the outputs of different firms.

Considering the second stage, suppose that total nominal expenditure on consumption has been chosen to be C^N . The consumer then solves

$$\max_{c(j)} n^{\frac{\mu}{\mu-1}} \left[\int_0^n c(j)^{1-\mu} dj \right]^{\frac{1}{1-\mu}} \quad (1.3)$$

subject to

$$\int_0^n p(j) \cdot c(j) dj \leq C^N \quad (1.4)$$

The solution to (1.3-1.4) is the demand for the output of each firm,

$$c(i) = \frac{p(i)^{\frac{-1}{\mu}}}{\int_0^n p(j)^{\frac{\mu-1}{\mu}} dj} C^N \quad (1.5)$$

The appropriate price index for subutility (1.1) can then be defined as

$$P = \left[\frac{1}{n} \int_0^n p(j)^{\frac{\mu-1}{\mu}} dj \right]^{\frac{\mu}{\mu-1}} \quad (1.6)$$

Note that, since (1.5) satisfies the budget constraint (1.4), $PC = C^N$. Hence using the quantity and price indices, (1.1) and (1.6) respectively, we are able to aggregate the outputs of the individual firms and treat them as a single (composite) commodity.

Returning to the first stage of the optimization process, the decision problem facing the household is

$$\max_{C, L^s, M} U \left(C, E - L^s, \frac{M}{P} \right)$$

subject to

$$P \cdot C + M \leq W \cdot L^s + \Pi + M^0 - T$$

the solution to which yields the standard Marshallian consumption and labour

supply functions

$$C = C\left(w, \frac{M^0 - T + \Pi}{P}\right); \quad C_1 > 0, C_2 > 0 \quad (1.7)$$

$$L^s = L^s\left(w, \frac{M^0 - T + \Pi}{P}\right); \quad L_1^s > 0, L_2^s > 0 \quad (1.8)$$

where $w = W/P$, the real wage, $C_1 = \partial C / \partial w$ and *etc.*

The indicated signs for the partial derivatives reflect the assumptions that each of the arguments of the utility function is a normal 'good' and that substitution effects outweigh income effects. The latter assumption ensures, of course, that labour supply is strictly increasing in the real wage, whilst the former implies a negative income or wealth effect on labour supply. In what follows we assume throughout that the functions C and L^s are twice-continuously differentiable.

1.2.2 The government

The government is assumed to formulate its expenditure plans in real terms. For any given price level this gives rise to a particular value of nominal expenditure, G , which is then allocated across firms according to government preferences, taken to be identical to those of the household sector. We assume the government to leave its potential monopsony power unexploited and hence its optimization process is entirely analogous to that of the household. In particular, the government chooses to purchase quantity $g(j)$ from firm j to solve:

$$\max_{g(j)} n^{\frac{\mu}{\mu-1}} \left[\int_0^n g(j)^{1-\mu} dj \right]^{\frac{1}{1-\mu}} \quad (1.9)$$

subject to

$$\int_0^n p(j) \cdot g(j) dj = G \quad (1.10)$$

As for household consumption the price index defined by (1.6) allows us to deflate nominal expenditure, to arrive at our representation of *real* government spending, that is $g = G/P$.

The finance of government expenditure may, in principle, be by means of lump-sum taxation, by money creation or by both used in combination, with the level of government spending and the means of finance linked by

the government budget constraint:

$$M - M^0 = G - T$$

In our analysis of fiscal policy in Section 1.4 we shall, in fact, focus exclusively on the effects of a balanced budget rise in government spending. This apparent limitation to the scope of our analysis is not so restrictive as it might seem, however; as will be explained, the real effects of a change in government expenditure are invariant to the means of finance of the policy.

1.2.3 The firm

Each firm $j \in [0, n]$ employs $l(j)$ units of labour to produce $y(j)$ units of its own variety of output. All firms share a common production technology described by

A2 : Technology: for all $j \in [0, n]$, we have

$$f : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \text{ such that } y(j) = f(l(j) - \lambda)$$

with f (at least) twice continuously differentiable, derivatives such that $f'' < 0 < f'$, $\lim_{l(j) \rightarrow \lambda} f' = \infty$ and $\lim_{l(j) \rightarrow \infty} f' = 0$. Thus, as in Startz, there is a fixed set up level of employment, λ , at or below which the level of output is zero. As employment increases beyond λ output expands but at a diminishing rate with successive increments of labour input; this assumption of a diminishing marginal physical product of labour represents one of the central differences between our own model and the SMD framework. The essential feature of the latter is the assumption of a constant marginal product of labour (CMPL) with $f'' = 0$ for $l \geq \lambda$. The SMD specification of production technology can thus be viewed as a special case of A2.⁶

Nominal profits of the firm are given by

$$\pi(j) = p(j) \cdot f(l(j) - \lambda) - W \cdot l(j) \quad (1.11)$$

The firm chooses $(p(j), l(j))$ to maximize (1.11) subject to its demand curve:

$$f(l(j) - \lambda) = \frac{1}{n} \left(\frac{p(j)}{P} \right)^{\frac{-1}{\mu}} (C + g) \quad (1.12)$$

In the maximization process the firm takes the *general* price level, P , as given and independent of its own actions; this is the essence of monopolistic competition in the Dixit-Stiglitz model. It is particularly attractive and plausible in the macroeconomic context, where P is the price index not of an industry but of the whole economy. Under such circumstances nominal

profit maximization seems the appropriate assumption to make.⁷ Clearly, treating as given exogenously means that (1.12) is a constant elasticity demand function, with elasticity $\varepsilon = 1/\mu$. The profit maximizing prices yields the mark-up of price, $p(j)$, over marginal cost, W/f' :

$$\frac{p(j) - \frac{W}{f'(l(j)-\lambda)}}{p(j)} = \frac{1}{\varepsilon} = \mu \quad (1.13)$$

Hence the parameter is equivalent to Lerner's index of monopoly (the price-cost margin). An alternative way of expressing (1.13) is in terms of the relationship between the marginal physical product of labour and the real wage. Defining the firms own-product real wage as $w(j) = W/p(j)$, we have:

$$w(j) = (1 - \mu) \cdot f'(l(j) - \lambda) \quad (1.14)$$

That is the firm chooses such that the real wage equals the marginal physical product of labour scaled down by $1 - \mu$; for the limiting case of perfect competition ($\mu \rightarrow 0$), the real wage and the marginal product are equated.

Since demand is symmetric across firms, each firm chooses the same price and employment level; hence $p(j) = P$ (implying $w(j) = w$) and $l(j) = l$. With employment per firm identical across firms aggregate employment, L , defined by:

$$L = \int_0^n l(j) dj$$

is given by $L = n \cdot l$. Similarly, aggregate output, Y , is related to output per firm, $y(j) = y$, in an obvious fashion, that is $Y = n \cdot y$. Real profits per firm are simply

$$\frac{\pi}{P} = y - w \cdot l$$

and with all firms earning identical profits, aggregate real profits are given by

$$\frac{\Pi}{P} = n \frac{\pi}{P} = n(y - w \cdot l) \quad (1.15)$$

1.2.4 Free entry and the firm in long-run equilibrium

With free entry (and exit) long-run equilibrium is characterized by the zero profit condition. This condition serves to tie down the long-run equilibrium values of the real wage and output and employment per firm. To see this note

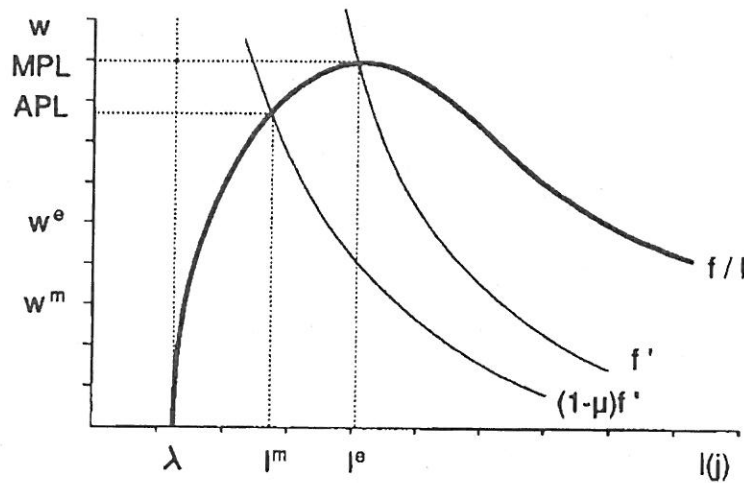


Figure 1.1: The firm in long-run equilibrium with free entry

that, since labour is the only input, profits are zero if and only if the real wage and the average product of labour are equal. Thus long-run equilibrium is characterized by

$$w^m = (1 - \mu) \cdot f'(l^m - \lambda) = \frac{f(l^m - \lambda)}{l^m} \quad (1.16)$$

which clearly determines fully the long-run monopolistic equilibrium values of employment, l^m (hence output $y^m = f(l^m - \lambda)$) per firm and the real wage, w^m .

The zero profit condition is represented diagrammatically in Figure 1.1 below. The 'efficient' scale of production for the firm occurs at the intersection of the inverted U-shaped average product of labour schedule (f/l) and the upper downward sloping curve, representing the marginal product of labour (f'). This intersection point corresponds to the long-run equilibrium position of the firm in the Walrasian case ($\mu = 0$), with employment per firm l^e , associated level of output y^e , and real wage w^e . For $\mu > 0$, however, the intersection point between the average product of labour schedule and the lower downward sloping curve, representing the marginal product of labour scaled down by $(1 - \mu)$ is the relevant one. Clearly the monopolistic long-run equilibrium occurs at a lower real wage and employment level, that is, $w^m < w^e$, $l^m < l^e$ for $\mu > 0$. In fact it is straightforward to see from the diagram that w^m , l^m and y^m are all decreasing in μ ; as μ increases, the lower curve, $(1 - \mu) \cdot f'$, is displaced vertically downwards producing an intersection point with the average product of labour schedule associated with smaller values of l and w . The difference between l^m and l^e is, of course, the

standard Chamberlinian excess capacity result; with monopolistic competition, the free entry long-run equilibrium is characterized by firms producing at below the optimal scale.

1.3 Equilibrium in a monopolistic economy

We now integrate the various components of the model outlined in the previous section in order to determine the characteristics of the short and long-run equilibria. The long-run equilibrium plays a central role in our analysis, due to the fact that it ties down the equilibrium real wage and hence output and employment per firm. Moreover, our policy analysis of Section 1.4 is conducted under the assumption that the economy begins from an initial position of long-run equilibrium. Accordingly we begin by exploring the properties of this equilibrium in some detail.

1.3.1 Long-run equilibrium

The economy comprises three markets; the markets for goods, for labour and for money. Walras' Law allows us to omit explicit consideration of the money market and focus our attention on the goods and labour markets. Long-run equilibrium is then characterized not only by the goods and labour market clearing conditions (Equations (1.17) and (1.18) below), but also by the free entry/zero profit condition (Equation (1.19))

$$C\left(w, \frac{M^0 - T + \Pi}{P}\right) + g = n \cdot f(1 - \lambda) \quad (1.17)$$

$$L^s\left(w, \frac{M^0 - T + \Pi}{P}\right) = n \cdot l \quad (1.18)$$

$$\begin{aligned} w &= (1 - \mu) \cdot f'(1 - \lambda) \\ &= \frac{f(1 - \lambda)}{l} \end{aligned} \quad (1.19)$$

The above system of equations implicitly defines n , l , w and P as functions of the exogenous parameters relating to household endowments, household preferences, production technology and government policy, that is, M^0 , μ , λ , g and T .

The recursive structure of the system is readily apparent; Equation (1.19) fully determines the levels of employment, output and profits per firm, $l = l^m$, $y = y^m$, $\Pi(l^m) = 0$ as well as the real wage, $w = w^m$. A solution to (1.19) exists under A2 for $0 \leq \mu < 1$, λ bounded, allowing us to re-express

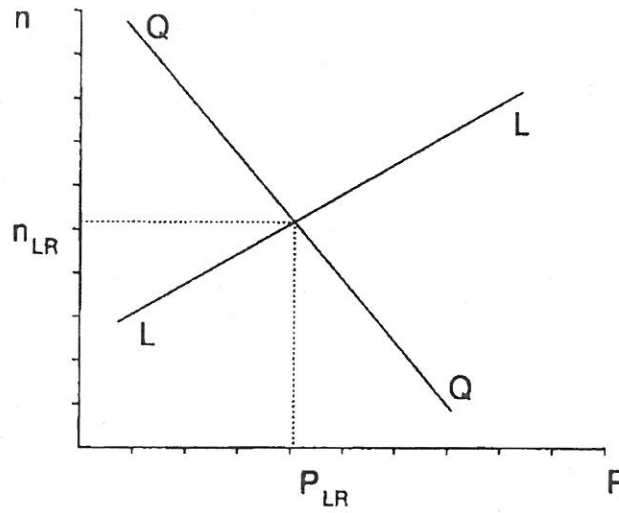


Figure 1.2: Long-run equilibrium

(1.17) and (1.18) as

$$C\left(w^m, \frac{M^0 - T}{P}\right) + g = n \cdot y^m \quad (1.20)$$

$$L^s\left(w^m, \frac{M^0 - T}{P}\right) = n \cdot l^m \quad (1.21)$$

This pair of equations now determines the long-run equilibrium values of the two endogenous variables, n and P . A diagrammatic representation of long-run equilibrium is provided in Figure 1.2. QQ represents goods market equilibrium and is downward sloping since, as the price level rises, real private sector wealth falls, thereby reducing consumption expenditure; consequently output must fall which, since output per firm is given in the long-run, requires a reduction in the number of firms, n .⁸ The gradient of the goods market clearing locus is given by

$$\left. \frac{dn}{dP} \right|_{QQ} = -C_2 \frac{M^0}{P^2} / y^m < 0$$

The labour market clearing locus is positively sloped, since the fall in real money balances which results from a higher price level increases desired labour supply; thus, given the fixed long-run level of employment per firm, labour market equilibrium requires a larger number of firms. The gradient of LL is

$$\left. \frac{dn}{dP} \right|_{LL} = -L_2^s \frac{M^0}{P^2} / l^m > 0$$

It is readily seen that money is neutral in this economy. Assuming T , like G , is fully indexed, it is clear that both (1.20) and (1.21) are homogeneous of degree zero in (M^0, P) . Hence an increase in the money supply will be reflected in a proportionate increase in all nominal magnitudes, leaving the values of all real variables undisturbed.⁹

In terms of Figure 1.2, a rise in M^0 leads to identical rightward horizontal shifts in the QQ and LL schedules, their intersection with respect to the vertical axis remaining unchanged.

An interesting and important issue is the nature of the dependence of the long-run equilibrium on the degree of monopoly, μ . Differentiation of (1.19) with respect to μ yields directly the influence of this parameter on the long-run values of the real wage, and output and employment per firm

$$\frac{dw^m}{d\mu} = \frac{\mu (f')^2}{l^m \cdot \psi} < 0 \quad (1.22)$$

$$\frac{dy^m}{d\mu} = \frac{(f')^2}{\psi} < 0 \quad (1.23)$$

$$\frac{dl^m}{d\mu} = \frac{f'}{\psi} < 0 \quad (1.24)$$

where $\psi = (1 - \mu) \cdot f'' - \mu \cdot f' / l^m < 0$.

Given our discussion of Figure 1.1 in the preceding section, the above results carry no surprises. A higher value of μ implies an increased wedge between the marginal product of labour and the real wage. Given the larger mark-up, the zero profit condition then implies a lower real wage and reduced levels of employment and output per firm.

Whilst it is clear from Equations (1.22)-(1.24) above that the influence of μ on w^m , y^m and l^m depends purely on technological factors, its impact on the number of firms, on aggregate output, Y , and employment, L , and on the price level, is determined by the system's general equilibrium characteristics. Differentiating Equations (1.20) and (1.21), using where appropriate (1.22)-(1.24), we find, after some straightforward manipulation¹⁰

$$\frac{dn}{d\mu} = \frac{[\mu (L_1^s C_2 - C_1 L_2^s) f' + L (L_2^s f' - C_2)] f'}{(l^m)^2 \psi (C_2 - w^m L_2^s)} \geq 0 \quad (1.25)$$

$$\frac{dY}{d\mu} = \frac{\mu [(L_1^s C_2 - C_1 L_2^s) w^m + L C_2] (f')^2}{l^m \psi (C_2 - w^m L_2^s)} \leq 0 \quad (1.26)$$

$$\frac{dL}{d\mu} = \frac{\mu [L_1^s C_2 - C_1 L_2^s + L L_2^s] (f')^2}{\psi (C_2 - w^m L_2^s)} \geq 0 \quad (1.27)$$

$$\frac{dP}{d\mu} = \frac{\mu [C_1 - w^m L_1^s - L] (f')^2 P^2}{l^m \psi (C_2 - w^m L_2^s) M^0} \geq 0 \quad (1.28)$$

Although, as indicated by Equation (1.25) the effect of μ on the total number of firms n is indeterminate, from (1.26) aggregate output is strictly decreasing in μ . This result is in accordance with that of Dixon (1987) and Mankiw (1988), though these papers are concerned with the properties of short-run equilibrium, that is, with the number of firms fixed. However, whilst the SMD assumption of a constant marginal product of labour implies directly that the inverse relationship between μ and aggregate output is reflected in a similarly negative relationship between μ and aggregate employment, in the present context the effect of μ on total employment can be seen from (1.27) to be ambiguous. Although, as noted above, employment per firm is negatively related to μ , the potential for a positive relationship between n and μ gives rise to the possibility that *aggregate* employment is increasing in μ . Of course, given the assumed production technology, this result is perfectly consistent with the finding of a negative relationship between total output and μ . Finally, it is apparent from (1.28) that the nature of the influence of μ on the price level is indeterminate. However two points are worth noting with regards to this result. First, the condition $C_1 - L < 0$ which is sufficient for $dP/d\mu > 0$ can be seen from (1.27) to be necessary for $dL/d\mu > 0$; given the real wage is negatively related to μ , a necessary condition for labour supply, and hence equilibrium employment to be increasing in μ , is a positive relationship between P and μ . Secondly, for the special case of Cobb-Douglas preferences, $C_1 - w^m L_1^s - L = (1 - \alpha - \beta) E < 0$, implying $dP/d\mu$ is unambiguously positive.

Before moving on to a discussion of short-run equilibrium within the model, we point to an interesting property of the relationship between the characteristics of long-run equilibrium and μ . That is, a small displacement of μ from its Walrasian value of zero has no effect on aggregate output, employment or the price level (1.26-1.28). The explanation for this result lies in the fact that in the Walrasian equilibrium each firm operates at the maximum point on its average product of labour curve. Consequently a small increase in μ from an initial value of zero and the implied fall in employment and output per firm have no first-order effect on average productivity and the long-run equilibrium real wage. Given the unchanged real wage, the requirement of simultaneous goods and labour market equilibrium then dictates that the decline in employment and output within each individual firm be precisely offset at the aggregate level by a compensating increase in the number of firms,¹¹ leaving the features of the long-run equilibrium otherwise undisturbed.

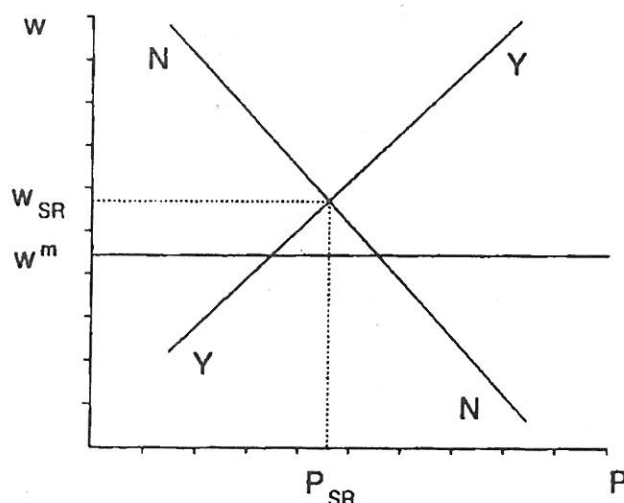


Figure 1.3: Short-run equilibrium

1.3.2 Short-run equilibrium

In the short run the number of firms is fixed. Equilibrium in the markets for goods and labour is described by (1.17) and (1.18) respectively, where of course, the real wage is given by $w = (1 - \mu) f'(l - \lambda)$. Thus with n constant, but employment per firm variable, the market-clearing conditions, together with the mark-up equation, can be viewed as determining the short-run equilibrium values of L , P and w .¹²

Figure 1.3 provides a diagrammatic representation of the short-run equilibrium and its relationship to the long-run steady state, depicting combinations of the price level and the real wage consistent with labour and goods market equilibrium, for given n . The horizontal schedule, LR , intersects the vertical axis at the value of the real wage, w^m , which must hold in long-run given the free entry condition. The negative slope of NN , representing equilibrium in the labour market, reflects the operation of the real balance effect on labour supply; as the price level rises labour supply increases, with the rise in employment necessary to equilibrate the labour market being reflected in a decline in the real wage.¹³ With regards to the goods market, however, an ambiguity is present. As the price level increases consumption falls via the real balance effect, but the change in the real wage necessary to maintain goods market equilibrium is indeterminate in direction.

In fact a sufficient condition for the goods market clearing locus YY to be upward sloping as depicted in Figure 1.3 is that the positive direct effect on consumption of a rise in the real wage outweighs the negative indirect effect which operates via the induced fall in profits, a condition which is

certainly fulfilled for Cobb-Douglas preferences. The gradient of the labour and goods market clearing loci are given by:

$$\left. \frac{dw}{dP} \right|_{NN} = \frac{-(1-\mu) f'' L_2^s \frac{M^0}{P}}{n(1-\mu f' L_2^s) - (1-\mu) f'' (L_1^s - LL_2^s)} < 0$$

$$\left. \frac{dw}{dP} \right|_{YY} = \frac{-(1-\mu) f'' C_2 \frac{M^0}{P^2}}{nf' (1-\mu C_2) - (1-\mu) f'' (C_1 - LC_2)} \geq 0^{14}$$

In the diagram NN and YY intersect at a value of the real wage above w^m . Accordingly, the depicted short-run equilibrium is associated with each existing firm earning negative profits and subsequently firms respond to these losses by ceasing production. Diagrammatically this decline in the number of operative enterprises is reflected in downward shifts of both NN and YY , leading to a fall in the short-run equilibrium real wage (but an ambiguous change in the price level) until the long-run steady state is achieved. In fact an adjustment process of this nature underlies the distinction between the short-run and long-run effects of fiscal policy to be discussed in the next section.¹⁵

1.4 Fiscal policy

In this section we examine the effects on the economy of an increase in government spending, focusing in particular on the short and long-run output and employment fiscal multipliers. Because our model encompasses the SMD framework as a special case we are able to provide straightforward comparisons between our own results and those of previous work on fiscal policy within an imperfectly competitive economy.

As already indicated we restrict our attention to balanced-budget fiscal policy; that is a rise in government spending financed entirely by an increase in lump-sum taxation. This has convenience value in that it allows us to avoid the need to take into account the monetary dynamics associated with the government budget constraint. At the same time it restricts the applicability of our analysis very little due to the fact that the real effects of a rise in government spending are independent of whether the means of finance is via taxation or by money creation. This feature is most easily seen by noting, from Equations (1.20) and (1.21),¹⁶ that following an increase in lump-sum taxation, T , equilibrium is restored simply by a fall in the price level sufficient in magnitude to return the household sector's non-labour income to its previous value. It follows that the only difference between the two alternative sources of finance of an expansionary fiscal policy derives from their

effects on the price level; thus the induced changes in output and employment, for example, are invariant to the means of finance of the policy.¹⁷

1.4.1 The short-run effects of fiscal policy

The effects of fiscal policy in the short run are found from (1.17) and (1.18), together with the mark-up relationship. Beginning from an initial position of long-run equilibrium (wherein $\Pi = 0$, $w = w^m$), differentiating these equations, holding n constant, and using Y and L to represent aggregate output and employment respectively, yields

$$\left. \frac{dY}{dg} \right|_{SR} = \Delta^{-1} > 0 \quad (1.29)$$

$$\left. \frac{dL}{dg} \right|_{SR} = \frac{1 - \mu}{w^m} \Delta^{-1} > 0 \quad (1.30)$$

$$\left. \frac{dw}{dg} \right|_{SR} = \frac{(1 - \mu)^2 f''}{n \cdot w^m} \Delta^{-1} < 0 \quad (1.31)$$

$$\left. \frac{dP}{dg} \right|_{SR} = -\frac{P^2}{M^0} \frac{1 - \mu}{n \cdot w^m L_2^s} \cdot \Upsilon \cdot \Delta^{-1} \geq 0 \quad (1.32)$$

where

$$\begin{aligned} \Upsilon &= n(1 - C_2 + w^m L_2^s) - (1 - \mu) f'' (L_1^s (1 - C_2) + L_2^s (C_1 - L)) \\ \Delta &= 1 - \frac{(1 - \mu) C_2}{w^m L_2^s} + \frac{(1 - \mu)^2}{n \cdot w^m} f'' \left(\frac{C_2 L_1^s}{L_2^s} - C_1 \right) > 1 \end{aligned} \quad (1.33)$$

The rise in government spending creates excess demand in the goods market. To restore equilibrium, an expansion of output (1.29) and, hence, employment (1.30) is necessary. But, with the number of firms fixed, the diminishing marginal product of labour implies, given the mark-up, a decline in the real wage (1.31) as employment increases. The requirement for goods market equilibrium is made compatible with equilibrium in the labour market via the rise in labour supply which results from the increase in lump-sum taxation, and by an appropriate change in the price level. The direction of adjustment of the price level is determined by whether an excess demand or excess supply of labour results from the change in employment dictated by considerations of goods market equilibrium together with the change in desired labour supply prompted by the increase in lump-sum taxation and the fall in the real wage.¹⁸

Given $\Delta > 1$ (1.33) it follows that the output multiplier is less than unity, which is in accordance with the results of previous analyses of balanced bud-

get fiscal policy within the SMD framework of an imperfectly competitive economy. It is straightforward to see precisely how a diminishing marginal product of labour affects the value of the multiplier compared with the SMD assumption of CMPL technology. In the latter case $f'' = 0$ of course, implying Δ is smaller in magnitude; hence the value of the output multiplier is larger. The reason lies in the fact that, with $f'' < 0$, as employment expands the real wage falls, reducing consumption expenditure directly and, at the same time, causing a fall in desired labour supply. This latter effect implies, *ceteris paribus*, a larger rise in the price level is required to equilibrate the labour market than is the case with a constant marginal product of labour resulting, via the real balance effect, in a larger fall in consumption spending. Hence, with $f'' < 0$ additional channels of crowding-out are present.

The precise value of the short-run output multiplier with a constant marginal product of labour is seen to be

$$\left. \frac{dY}{dg} \right|_{SR} = \frac{1}{1 - \frac{1-\mu}{w^m} \frac{C_2}{L_2^s}} \quad (1.34)$$

Further, adopting the assumption of Cobb-Douglas preferences allows us to obtain an expression for the output multiplier which is directly comparable with the DSM results:

Proposition 1.1 *Assume Cobb-Douglas preferences, with utility function $U = C^\alpha (E - L^s)^\beta (M/P)^\gamma$, and CMPL technology, $f'' = 0$. Then:*

$$\left. \frac{dY}{dg} \right|_{SR} = \frac{\beta}{\beta + (1 - \mu) \alpha}$$

The value of the multiplier given by the above expression is identical to that of Dixon and equivalent to those contained in Mankiw and Startz.¹⁹ For this special case it is readily apparent that the magnitude of the multiplier is increasing in μ . This particular result, which rests upon the dependence on μ of the relationship between aggregate profits (thereby disposable income) and aggregate output, is the feature which lends the Keynesian flavour to analyses of fiscal policy in models of imperfectly competitive economies. However, inspection of (1.33) indicates that Δ is related to μ in a highly complex fashion; in general, the nature of this relationship will depend on the various second-order derivatives, C_{ij} , L_{ij}^s , the precise form of the implicit functional dependence of both w^m and P on μ , and on the sign and magnitude of the third derivative of the production function. Thus, there is no general presumption that the size of the short-run output multiplier is increasing in μ .

1.4.2 The impact of fiscal policy in the long run

In the short run, with the number of firms fixed, the expansion in output and employment associated with a rise in government spending leads to an increase in profits from their long-run equilibrium value of zero. The prospect of positive profits prompts the entry of new firms until the inducement to enter has itself been eliminated, that is, profits have returned to zero. This expansion in the number of firms, of course, changes the characteristics of the equilibrium and, hence, modifies the impact of fiscal policy compared to the short run. We find the effects of fiscal policy in the long run by differentiation of (1.20) and (1.21) and solving for the changes in the endogenous variables n and P ; the adjustments in aggregate output and employment are then found using $Y = ny^m$ and $L = nl^m$.

$$\left. \frac{dY}{dg} \right|_{LR} = \frac{1}{1 - \frac{1}{w^m} \frac{C_2}{L_2^s}} > 0 \quad (1.35)$$

$$\left. \frac{dL}{dg} \right|_{LR} = \frac{1}{w^m - \frac{C_2}{L_2^s}} > 0 \quad (1.36)$$

$$\left. \frac{dn}{dg} \right|_{LR} = \frac{1}{y^m \left(1 - \frac{1}{w^m} \frac{C_2}{L_2^s} \right)} > 0 \quad (1.37)$$

$$\left. \frac{dP}{dg} \right|_{LR} = \frac{P^2}{M^0} \frac{(1 - C_2 + w^m L_2^s)}{(C_2 - w^m L_2^s)} > 0 \quad (1.38)$$

With the real wage and output and employment per firm tied down by the zero profit condition, the increase in aggregate output (1.35), and associated rise in employment (1.36), necessary to maintain goods market equilibrium following the fiscal expansion are achieved purely by way of an increase in the number of firms (1.37). To induce the required rise in labour supply, the price level must increase (1.38) which, in turn, crowds out some private sector consumption. Hence the long-run output multiplier, like the short-run multiplier, is less than one in value.

Just as the relationship between μ and the magnitude of the short-run multiplier is indeterminate in direction, so is that between μ and the value of the long-run multiplier. However, an interesting result emerges for the special case of Cobb-Douglas preferences, where the long-run multiplier (1.35) becomes²⁰

$$\left. \frac{dY}{dg} \right|_{LR} = \frac{\beta}{\beta + \alpha} \quad (1.39)$$

Thus for Cobb-Douglas preferences it is clear that the long-run multiplier is *independent* of μ . This result generalizes that of Startz (pp. 748-749), obtained under the assumption of CMPL technology:

Proposition 1.2 *If household preferences are Cobb-Douglas in form then, for any technology satisfying A2, the long-run output multiplier, given by (1.39), is independent of the degree of monopoly, μ .*

Although the result summarized in Proposition 1.2 is clearly a special one, the independence from μ of the value of the long-run multiplier extends to more general preferences in the neighbourhood of the Walrasian equilibrium. That is:

Proposition 1.3 *For any preferences satisfying A1 and any technology satisfying A2, then a small increase in μ from its Walrasian value of zero has no first order effect on the long-run output and employment multipliers.*

This proposition is straightforward to prove by differentiation of equation (1.35) and (1.36) with respect to μ ²¹ and follows directly from the fact that in the neighbourhood of Walrasian equilibrium the long-run levels of output and employment remain invariant to a small displacement of μ from zero. Note also that the local result of Proposition 1.3 relates to the employment as well as the output multiplier, whilst the global Proposition 1.2 applies only to the latter. In fact, with Cobb-Douglas preferences, the long-run employment multiplier is given by

$$\frac{dL}{dg} = \frac{1}{w^m} \frac{\beta}{\beta + \alpha}$$

which is increasing in μ for $\mu > 0$.²²

1.4.3 Comparing the short and long-run output and employment multipliers

Given the distinction between equilibrium in the short and long runs and our discussion of the corresponding multipliers, a natural issue to examine is the question of whether fiscal policy is more powerful in the short run or in the long run. Within the SMD framework only Startz addresses this question; his findings are reflected in the results summarized in Propositions 1 and 2

$$\left. \frac{dY}{dg} \right|_{LR} = \frac{\beta}{\beta + \alpha} < \frac{\beta}{\beta + (1 - \mu)\alpha} = \left. \frac{dY}{dg} \right|_{SR}$$

for $0 < \mu < 1$. That is, with CMPL technology and Cobb-Douglas preferences, then for $\mu \in (0, 1)$ the output multiplier is greater in the short run than in the long run. In fact, a comparison of (1.34) with (1.35) provides a rather more general result, summarized in Proposition 1.4:

Proposition 1.4 *Given CMPL technology and with $0 < \mu < 1$, then for any preferences satisfying A1*

$$\left. \frac{dY}{dg} \right|_{LR} = \frac{1}{1 - \frac{1}{w^m} \frac{C_2}{L_2^s}} < \frac{1}{1 - \frac{1-\mu}{w^m} \frac{C_2}{L_2^s}} = \left. \frac{dY}{dg} \right|_{SR}$$

Thus the assumption which underpins Startz's ranking of short and long-run output multipliers is seen to be that of CMPL technology; the structure of household preferences is clearly irrelevant for this result. Nonetheless the technological specification adopted by Startz is rather a special case. Comparing (1.29) and (1.35) we find:

$$\left. \frac{dY}{dg} \right|_{SR} \geq \left. \frac{dY}{dg} \right|_{LR} \text{ as } \frac{\mu}{(1-\mu)^2} \geq \frac{f''}{n} \left(\frac{C_1 L_2^s}{C_2} - L_1^s \right) \quad (1.40)$$

and, in general, the direction of this inequality is indeterminate. An important result emerges however for the Walrasian case of $\mu = 0$; in this case it is apparent from (1.40) that the long-run multiplier is greater in magnitude than the short-run multiplier. Comparing (1.29) for $\mu = 0$ with (1.35) we have:

$$\begin{aligned} \left. \frac{dY}{dg} \right|_{SR(\mu=0)} &= \frac{1}{1 - \frac{1}{w^m} \frac{C_2}{L_2^s} + \frac{f''}{n \cdot w^m} \left(\frac{C_2 L_1^s}{L_2^s} - C_1 \right)} \\ &< \frac{1}{1 - \frac{1}{w^m} \frac{C_2}{L_2^s}} = \left. \frac{dY}{dg} \right|_{LR(\mu=0)} \end{aligned}$$

The explanation for this ranking of the Walrasian short and long-run output multipliers lies in the fact that, with $f'' < 0$, the short-run response to the rise in government spending involves an expansion of firms beyond the efficient scale, where production is located in the Walrasian long-run equilibrium. Subsequently, as the prospect of positive profits induces the entry of new firms, the associated increase in the average product of labour allows an expansion of aggregate output even in the absence of any increase in labour supply. In fact, the rise in the real wage, which accompanies the rise in the *marginal* product of labour as new firms enter, leads to an increase in both labour supply and consumption expenditure. Thus in the Walrasian

case there is less crowding out in the long-run than in the short-run. Our Walrasian results are summarized in Proposition 1.5:

Proposition 1.5 *In the Walrasian case of $\mu = 0$:*

$$0 < \left. \frac{dY}{dg} \right|_{SR(\mu=0)} < \left. \frac{dY}{dg} \right|_{LR(\mu=0)} < 1$$

The significance of Proposition 1.5 is that it reverses the ranking of the short and long-run multipliers found by Startz for $\mu > 0$ and CMPL technology. In fact Proposition 1.5 can be generalized somewhat. Assuming the conditions of the Implicit Function Theorem are satisfied, both the short and long-run multipliers will be continuous in μ in the neighbourhood of $\mu = 0$. Hence for values of μ sufficiently close to zero the direction of the inequality in Proposition 1.5 will be maintained and we have:

Proposition 1.6 *A1, A2. If the derivatives f'' , C_1 , L_1^s , C_2 , L_2^s are continuous in the neighbourhood of the Walrasian equilibrium, then there exists $\bar{\mu} > 0$, such that for $0 \leq \mu < \bar{\mu}$,*

$$\left. \frac{dY}{dg} \right|_{SR} < \left. \frac{dY}{dg} \right|_{LR}$$

Thus Proposition 1.6 emphasizes that Startz's ranking of the short and long-run output multipliers is not at all a general result, but instead a reflection of the rather special assumptions which characterize the SMD framework and, in particular, that of CMPL technology.

Although an unambiguous comparison of the short and long-run *output* multipliers can be made only in the neighbourhood of the Walrasian equilibrium, a general ranking of the corresponding employment multipliers is possible. From Equations (1.30) and (1.36) we find:

$$\begin{aligned} & \left. \frac{dL}{dg} \right|_{LR} - \left. \frac{dL}{dg} \right|_{SR} \\ &= \left(\frac{\mu w^m}{1-\mu} + \frac{1-\mu}{n} f'' \left(\frac{C_2 L_1^s}{L_2^s} - C_1 \right) \right) \Delta^{-1} \left(w^m - \frac{C_2}{L_2^s} \right)^{-1} > 0 \end{aligned}$$

Therefore we have:

Proposition 1.7 *For all $\mu \in [0, 1)$,*

$$\left. \frac{dL}{dg} \right|_{SR} < \left. \frac{dL}{dg} \right|_{LR}$$

Thus despite the possibility of a larger output multiplier in the short run than in the long run, the long-run employment multiplier is *necessarily* greater in magnitude than the short-run employment multiplier. This ranking of the employment multipliers follows from the rise in the marginal product of labour which accompanies the entry of new firms. The associated increase in the real wage induces an expansion of labour supply and hence employment. The fixed overhead labour requirement, which implies, for $\mu > 0$, the average product falls with new entry, means that this increase in employment remains compatible with the potential fall in aggregate output.

1.5 A diagrammatic exposition of the fiscal multiplier with imperfect competition

1.5.1 The short-run relationship: results

Whilst the formal results have been shown for a monetary economy with a general production function, we can illustrate the results in a non-monetary economy (as in Dixon and Lawler, 1996). The only restriction on preferences we make for this diagrammatic analysis is that preferences U are homothetic. With homothetic preferences, the marginal rate of substitution between leisure and consumption depends only on the ratio $C/(E - L)$, where E is the endowment of leisure: utility maximization implies that the ratio of these two is therefore a function of the relative price w :

$$\frac{C}{E - L} = \gamma(w) \quad (1.41)$$

where γ is strictly decreasing in w , $\gamma' < 0$, since consumers have a strictly decreasing marginal rate of substitution. The Income Expansion Path (IEP) of consumers are therefore linear, and depend only on the relative price (real wage) w .

If we assume that there are CRTS, then we can normalize the MPL to unity, so that $w = 1 - \mu$. Hence, as μ increases, the income expansion in $(C, E - L)$ space becomes flatter. This reflects the fact that more imperfect competition leads to the price of consumption rising relative to that of leisure (w falls), and hence households substitute away from consumption to leisure, as depicted in Figure 1.4: the Walrasian IEP corresponds to $w = 1$ ($\mu = 0$), and more imperfectly competitive to higher value of μ .

Now we can turn to the production side. Leaving aside fixed costs, we can represent the production possibility frontier in $(C, E - L)$ space. The

PPF for the economy is represented by:

$$L - C - g = 0 \quad (1.42)$$

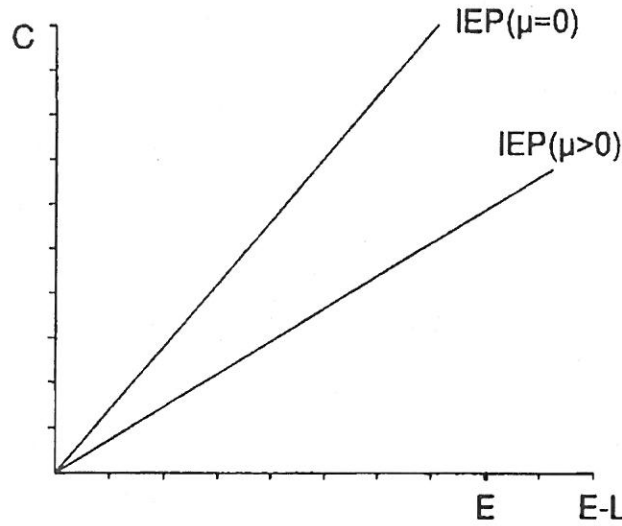


Figure 1.4: Income expansion paths

Hence, in $(C, E - L)$ space, we have a series of negatively sloped 45° lines: those closer to the origin corresponding to higher levels of g , as in Figure 1.5. The need to supply higher levels of g reduces the possibilities for private use. We can now put together Figures 1.4 and 1.5, and see the impact of imperfect competition the multiplier. In Figure 1.6, we depict an initial equilibrium for two values of μ : the Walrasian and an imperfectly competitive value. Turning first to the Walrasian: when $\mu = 0$, the equilibrium occurs at point W , and the marginal rate of substitution equals the Marginal Rate of Transformation (MRT) of leisure into consumption (MRT=1). The household budget constraint corresponds to the PPF. This is apparent, because when $\mu = 0$, constant returns to scale implies zero profits; hence the household budget constraint is:

$$C = wL - T \quad (1.43)$$

with a balanced budget $T = g$, and when $\mu = 0$, $w = 1$, so that (1.43) is equivalent to (1.42).

Now let us turn to the imperfectly competitive case. Here, the MRS equals $-(1 - \mu)$, which is less than the MRT in absolute terms: giving up one unit of leisure appears to increase consumption by only $(1 - \mu)$. The equilibrium occurs at point M in Figure 1.6, where the PPF intersects the $IEP(\mu)$. The perceived budget constraint passes through M , with corresponding con-

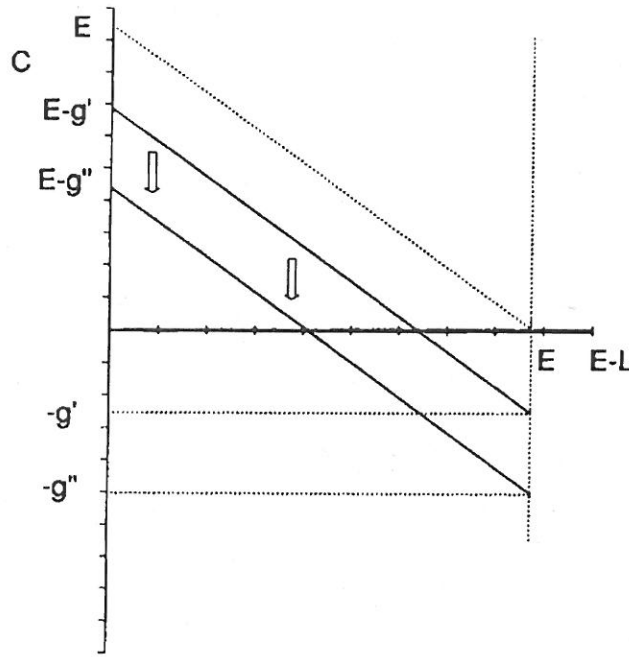


Figure 1.5: The PPF for consumption and leisure as g varies

sumption C_M . The vertical distance OC_0 represents the consumption level when the household supplies no labour ($L = 0$, leisure equals E). This represents the profits of firms which are redistributed to the household less tax T . In general, when $\mu > 0$ we have the budget constraint (for any C):

$$C = wL + \Pi - g \quad (1.44)$$

where $\Pi = \mu \cdot (C + g)$ and $w = 1 - \mu$ (note that the household treats Π as fixed, and the relation between Π and C comes from the firm's budget constraint). Clearly, at point M , $C = C_M$, and the budget constraint can be written as:

$$C = \mu \cdot (C_M + g) - g = \Pi - g \quad (1.45)$$

At point M , the budget constraint passes through the PPF (since $L = C_M + g$). However, at all other points this is not true. At $L = 0$, we have the corresponding consumption C_0 , where:

$$C_0 = \mu \cdot (C_M + g) - g = \Pi - g \quad (1.46)$$

Now we consider the effect of an increase in g by Δg , which shifts the PPF in $(C, E-L)$ space down by the vertical distance Δg . We can see this in Figure 1.7, where the new equilibria are W' and M' respectively. If we define total

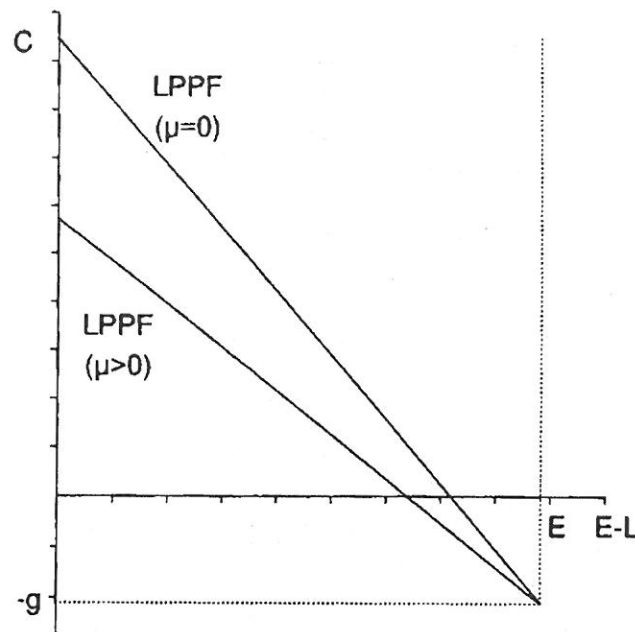


Figure 1.8: The long-run production possibility frontiers

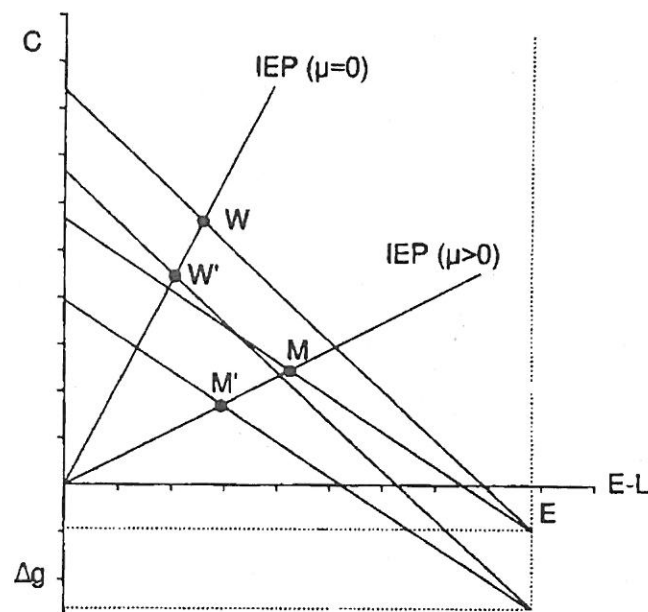


Figure 1.9: The long-run multipliers compared

if more output is produced, it can be produced more efficiently than along the LPPF (since the APL of labour is increasing), up to point B^W on the Walrasian LPPF (at that point APL is maximized). There after the SPPF passing through and B' will lie within the Walrasian LPPF, and may (or may not) intersect the LPPF($\mu > 0$). Starting from B , if less output is produced, the SPPF lies inside the LPPF($\mu > 0$).

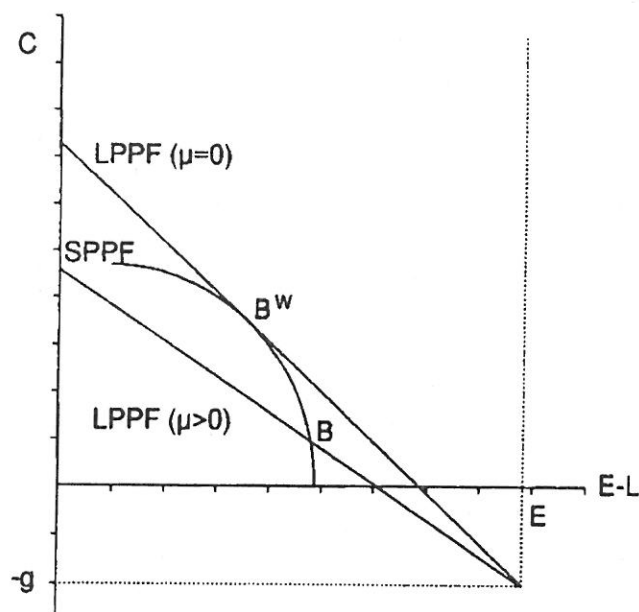


Figure 1.10: The short-run and long-run multipliers compared when $\mu > 0$

Now, consider an increase in g : this will shift the SPPF and the LPPF passing through B down to B' , as in Figure 1.11. The long-run effect is a move down the same IEP to point C . However, in the short run, the move will be along the new SPPF which passes through B' . As drawn, the SPPF passing through B' lies above C : that is the new long-run equilibrium is within the old SPPF. This is not always the case: although the segment of the LPPF immediately to the north-west of B' will lie with the SPPF, it is quite possible that the SPPF will intersect the LPPF at a point between B' and C . When the real wage will decline as we move along the SPPF (although profit inclusive income increases): the relative price of consumption/leisure is $w = (1 - \mu)f'$. Since consumption appears more expensive, we will switch to an IEP which intersects the LPPF between B' and C . For example, in the Walrasian case when $\mu = 0$, the SPPF is tangential to the LPPF at B' , and hence C lies outside the SPPF: this will also be true for μ small enough. If we look at Figure 1.11, we can draw a horizontal line through C . As drawn, it intersects the SPPF at D . Clearly, if the new short run lies below D , then the degree of crowding out is larger in the short run than in the long run: conversely, if the new short-run equilibrium lies above D , then the degree of crowding out in the short run must be less. Thus, it is possible for the short-run multiplier to be larger or smaller than the long-run. Since Startz looked at a case where the short-run multiplier exceeded the long-run, it is easy to see how this might be reversed: if μ is small enough, then the SPPF lies everywhere below the horizontal line passing through C , so that

the short-run multiplier must be less than the long-run (as was demonstrated in Proposition 1.6).

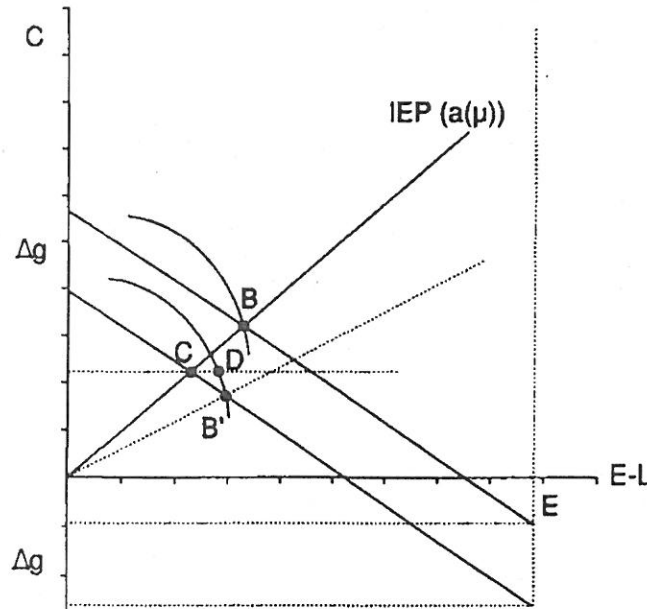


Figure 1.11: The short-run and long-run multipliers compared when $\mu > 0$

1.6 Dynamic equilibrium models

In this section, we will show how the results of the previous section are carried over into a dynamic context. We will adopt the continuous time Ramsey case, with an infinitely lived consumer. A full analysis is given in Dixon (1997), but here we will deal with steady-state effects of fiscal policy, under the assumption of instantaneous free entry. The flow utility function of the consumer depends on aggregate consumption $C(t)$ and leisure $1 - L(t)$. The production of the representative firm is given by the production function of the form (we suppress the firm subscript):

$$y(t) + \varphi = F(K(t), L(t)) \quad (1.51)$$

where $y(t)$ is output and φ is the overhead fixed cost per firm. With constant elasticity of demand for output, monopolistic firms will mark up price over marginal cost. We now have two factor demand equations implied by this:

$$w = (1 - \mu) F_L; \quad r = (1 - \mu) F_K \quad (1.52)$$

where r is the real interest rate. If we assume that F is homogeneous to degree 1, then we can write F in intensive form: $y(t) + \varphi = L(t) \cdot f(k(t))$

where $k(t) = K(t)/L(t)$, and $F_K = f'$ and $F_L = f' - k \cdot f'$. By Euler's equation, since F is homogeneous, $F = L \cdot F_L + K \cdot F_K$, so that the profits of the firm are, using (1.51):

$$\Pi(t) = \mu \cdot F - \varphi = \mu \cdot y(t) - (1 - \mu) \varphi \quad (1.53)$$

Now, if we assume that there is free entry, then profits will be zero. As in the case where there was one factor of production, this implies that the output per firm is tied down:

$$y(t) = \left(\frac{1 - \mu}{\mu} \right) \varphi \quad (1.54)$$

Aggregating over firms, the total number of firms $n(t)$ is given by the relationship:

$$n(t) = \frac{Y(t)}{y(t)} = Y(t) \cdot \left(\frac{\mu}{1 - \mu} \right) \frac{1}{\varphi} \quad (1.55)$$

Hence, the free entry condition implies that number of firms varies as the aggregate output varies. We can combine (1.51) and (1.55), to obtain the aggregate production function given that there is imperfect competition and free entry:

$$Y(t) = (1 - \mu) \cdot F(K(t), L(t)) \quad (1.56)$$

This equation is very important: it means that although the technology per firm displays increasing returns to scale ($\varphi > 0$), the aggregate technology displays constant returns due to free entry. This would apply so long as F is homothetic, and does not require F to be homogeneous to degree 1. Note that in the case of F being homogeneous to degree one or more, we have a natural monopoly, and that entry of additional firms is inefficient. Imperfect competition increases the inefficiency due to excess entry, since when μ is larger it raises profitability per firm for a given number of firms.

Given this, we can write down the constrained social planner's problem which yields the imperfectly competitive economy:

$$\begin{aligned} \max \int_0^\infty e^{-\beta \cdot t} \cdot U(C(t), 1 - L(t)) dt \\ \text{s.t. } \frac{dK}{dt} = (1 - \mu) F(K(t), L(t)) - C(t) - \delta K(t) - g(t) \end{aligned} \quad (1.57)$$

In formulating the optimization, we are taking the free entry condition and imperfect competition as given, and thus solving the second-best problem. The first best would involve freely choosing the number of firms and output

per firm.²³ The current value Hamiltonian for (1.57) is:

$$H(t) = U(C(t), 1 - L(t)) + \lambda(t) ((1 - \mu) F(K(t), L(t)) - C(t) - \delta K(t) - g(t)) \quad (1.58)$$

The first order conditions for this are:

$$H_C = U_1 - \lambda = 0 \quad (1.59)$$

$$H_L = -U_2 + \lambda(1 - \mu) F_L = 0 \quad (1.60)$$

$$H_K = \lambda((1 - \mu) F_K - \delta) = -\frac{d\lambda}{dt} + \beta\lambda \quad (1.61)$$

with the transversality condition and $H_\lambda = 0$. Equations (1.59-1.61) define the dynamic equilibrium of the imperfectly competitive economy. If we are concerned with the steady state only, then we can write these in the familiar form (using the intensive form production function):

$$(1 - \mu) f' = \delta + \beta \quad (1.62)$$

$$(1 - \mu) (f(k) - k \cdot f') = \frac{U_L}{U_C} \quad (1.63)$$

Equation (1.62) is the intertemporal optimality condition, sometimes called the modified golden rule (MGR); (1.63) is the intratemporal optimality condition equating the MRS between consumption and leisure with the real wage.

Clearly, the solution to (1.59-1.61) can define the entire dynamics of the system given $g(t)$: for a full analysis see Dixon (1997). Since in this paper we are only interested in the steady state effects, we will consider the steady state fiscal multiplier: the multiplier that occurs when there is an unanticipated permanent change in g from one level to another. This is easily represented diagrammatically. The MGR (1.62) can be solved for the optimal capital-labour ratio, $k^*(\mu)$, with $k^{*'} < 0$. This then defines the optimal net output per unit labour $(1 - \mu) \cdot f(k^*(\mu)) - \delta k^*(\mu)$. Hence, we can represent the MGR in $(C, 1 - L)$ space, given g . It is the line which we call the intertemporal PPF (IPPF) defined by:

$$C = L((1 - \mu) \cdot f(k^*(\mu)) - \delta k^*(\mu)) - g \quad (1.64)$$

The IPPF defines the trade off between leisure and consumption in the steady state given that accumulation satisfies the MGR.²⁴ Note that as μ increases, the curve rotates anticlockwise from the point $(-g, 1)$. The effect of imperfect competition is twofold: first, it reduces the returns to investment,

yielding less saving/investment, and hence the MGR yields a lower steady state $k^*(\mu)$; secondly the inefficiency due to excess entry is increased. Furthermore, with free entry the equilibrium level of productivity is lower, so that for any given k , output per unit labour is lower. The intratemporal optimality condition is exactly the same as in (1.41), and can be represented by the IEP $\gamma(w)$, where now:

$$w(\mu) = (1 - \mu) \cdot (f(k^*(\mu)) - k^* \cdot f'(k^*(\mu))) \quad (1.65)$$

Note that μ reduces the steady state real wage (1.65) in two ways: first $k^*(\mu)$ falls, and second $(1 - \mu)$ falls. The Walrasian equilibrium only exists when $\mu = \varphi = 0$, since when $\varphi > 0$ there are increasing returns to scale.

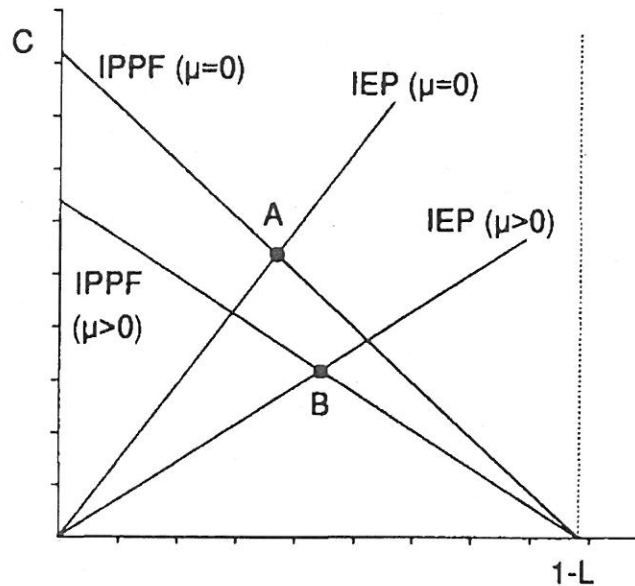


Figure 1.12: The Walrasian (A) and imperfectly competitive (B) equilibria

However, even when $\varphi > 0$, an equilibrium exists for all $\mu > 0$, and in this case we can rather loosely take the Walrasian economy as the limiting equilibrium as $\mu \rightarrow 0$. In Figure 1.12 we compare this limiting Walrasian economy ($\mu = 0$) with the imperfectly competitive economy: the Walrasian equilibrium is at point A, and the imperfectly competitive at point B. If we compare Figure 1.12 to Figure 1.6, the two look very similar. This is not surprising: in both cases there is a negative linear relationship between consumption and leisure. In the static case this was due to free entry; in the dynamic case, it is due to free entry combined with the intertemporal optimality condition. However, the visual similarities hide some important differences. First, the slope in the static case is equal to the real wage: this is not so in the dynamic case - the slope of the IPPF includes income from

wages $(1 - \mu) \cdot [f(k^*(\mu)) - k^*(\mu) \cdot f'(k^*(\mu))]$ and from the rental on capital $(1 - \mu) \cdot k^*(\mu) \cdot f'(k^*(\mu)) - \delta k^*(\mu)$ which combine to make the total steady-state income per unit labour of $(1 - \mu) \cdot f(k^*(\mu)) - \delta k^*(\mu)$.

Let us first consider the steady-state effects of a permanent increase in g , as depicted in Figure 1.13. This will lead to the IPPF to shift downwards by the distance dg . As a result, both consumption and leisure will fall: this yields a multiplier which is less than one (since consumption falls), but greater than zero (since total output rises as leisure falls). Clearly, the household is made worse off because it has to pay the taxes to fund dg ; it responds to this reduction in utility by reducing both leisure and consumption. Output rises, but by less than the increase in g . The story is almost exactly the same as in the static case, as is reflected in the similarity of Figures 1.13 and 1.7.

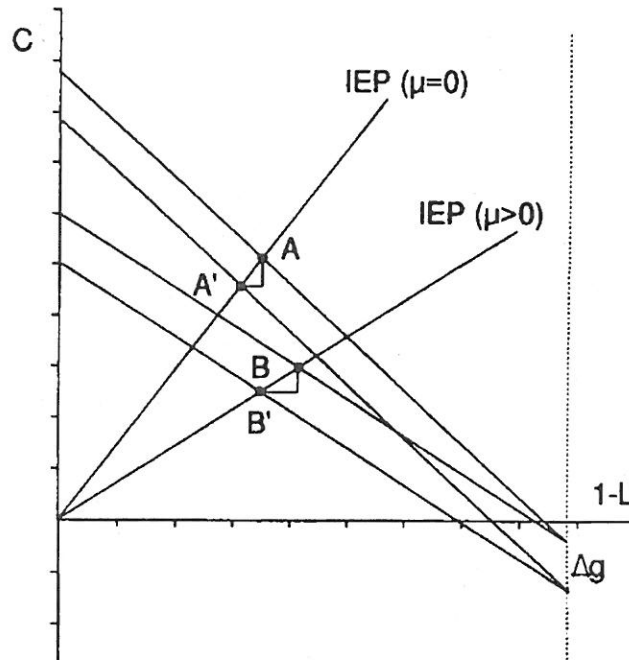


Figure 1.13: The Walrasian multiplier ($A - A'$) and the imperfectly competitive multiplier ($B - B'$)

How is the multiplier affected by an increase in μ ? Here, the IEP will become flatter, as will the IPPF. In Figure 1.13 we compare the multiplier for two different values of μ . There are two counteracting tendencies here. First, as μ increases the IEP becomes flatter: this means that the reduction in leisure is greater and the reduction in consumption smaller. This tends to make the employment multiplier larger, and the output multiplier smaller. Second, the IPPF also becomes flatter. This second effect tends to increase the reduction in consumption and leisure. Thus the combined effect is unam-

biguous for the employment multiplier: it is larger when μ is larger (confirming Proposition 1.7 in the static case). However, the effects on consumption, the crowding-out effects, are ambiguous: the fall in consumption could be larger or smaller, depending on the specific assumptions about functional forms.

In this section we have developed a simple analysis of the dynamic intertemporal model without the need to assume any explicit functional forms. As we have seen, if we restrict ourselves to steady state reactions to permanent changes in government expenditure, the analysis is very similar to the static case analyzed in Dixon and Lawler (1996) and the previous sections of this paper. For a continuous time intertemporal model similar to the one presented in this section, which extends the analysis to the short-run dynamics, see Heijdra (1997). However, the analysis of intertemporal models with imperfect competition has tended to be done in a discrete time RBC framework (see for example Rotemberg and Woodford, 1995, Devereux *et al.*, 1996). This approach often ends up using explicit functional forms, and it becomes unclear what results are general, and which are driven by the explicit functional forms. Whilst adopting explicit functional forms allows the calibration of the model and its confrontation with the data, it also can dramatically restrict the range of possible responses.

1.7 Conclusion

In this paper we have generalized existing models which seek to analyze fiscal policy within the context of an economy with monopolistic output markets and a Walrasian labour market. Our approach highlights the dangers of attempting to draw too general conclusions from models which make specific assumptions about functional forms. The main implications of the analysis of this paper are two-fold. First, the relationship between imperfect competition and the behaviour of the macroeconomy is not as simple as suggested by the SMD framework. Rather the results found within the latter rest on one or both of the crucial assumptions of constant marginal expenditure shares and a constant marginal product of labour. Secondly, whilst matters are not surprisingly more ambiguous in our own framework, we can still extract some general results:

1. if the economy is sufficiently competitive then the long-run output multiplier exceeds the corresponding short-run multiplier, with both lying between zero and unity.
2. whatever the degree of monopoly, including the Walrasian limit, the

long-run employment multiplier is greater than that obtaining in the short-run.

3. starting from the Walrasian limit, a small increase in the degree of monopoly has no first order effect on either the long run levels of output and employment or the corresponding long-run multipliers.

Thirdly, dynamic intertemporal models often make very restrictive assumptions about preferences and technology. These assumptions will tend to determine the results of these models as in the static framework, and perhaps we should not treat the results of these dynamic models as anything other than rather special cases. Just as in Dixon and Lawler (1996) and the present paper have generalized the static model of SMD, there is an urgent need to allow for much more general preferences and technology in dynamic models. Fourthly, we would strongly argue for the need to develop a visual and geometric understanding of macroeconomic models.

The existence of some general results in this paper is encouraging. It appears to indicate the possibility of moving beyond the highly specific models which typify the macroeconomics of imperfect competition literature without losing the capability to obtain useful results. Let us hope that researchers in the coming years are able to develop models in this way.

Appendix

1. Long-run equilibrium and μ

Differentiating (1.20) and (1.21) with respect to μ , noting that, with T fully indexed, the real value of taxation is independent of μ :

$$C_1 \frac{dw^m}{d\mu} - \frac{M^0}{P} C_2 \frac{dP}{d\mu} = n \frac{dy^m}{d\mu} + y^m \frac{dn}{d\mu} \quad (1.66)$$

$$L_1^s \frac{dw^m}{d\mu} - \frac{M^0}{P} L_2^s \frac{dP}{d\mu} = n \frac{dl^m}{d\mu} + l^m \frac{dn}{d\mu} \quad (1.67)$$

Substituting the values for $\frac{dw^m}{d\mu}$, $\frac{dy^m}{d\mu}$, and $\frac{dl^m}{d\mu}$ given by equations (1.22)-(1.24) of the main text yields, after some rearrangement:

$$\begin{bmatrix} y^m & \frac{M^0}{P} C_2 \\ l^m & \frac{M^0}{P} L_2^s \end{bmatrix} \begin{bmatrix} \frac{dn}{d\mu} \\ \frac{dP}{d\mu} \end{bmatrix} = \begin{bmatrix} \frac{(\mu C_1 - L)(f')^2}{l^m \psi} \\ \frac{(\mu L_1^s f' - L)f'}{l^m \psi} \end{bmatrix} \quad (1.68)$$

The solution to (1.68) provides expressions for (1.25) and (1.28). The effect of μ on aggregate output (1.26) and employment (1.27) can then be found in an obvious way using $Y = ny^m$, $L = nl^m$.

2. Fiscal policy and long-run equilibrium

We differentiate (1.20) and (1.21) with respect to g , noting that w^m , y^m and l^m are all invari-

ant to the stance of fiscal policy, whilst, given the balanced budget nature of the expansion, $d(T/P)/dg = 1$:

$$\begin{bmatrix} y^m & \frac{M^0}{P} C_2 \\ l^m & \frac{M^0}{P} L_2^s \end{bmatrix} \begin{bmatrix} \frac{dn}{dg} \\ \frac{dP}{dg} \end{bmatrix} = \begin{bmatrix} 1 - C_2 \\ -L_2^s \end{bmatrix} \quad (1.69)$$

Solving (1.69) yields $dn/dg|_{LR}$ (1.37) and $dP/dg|_{LR}$ (1.38) directly, allowing the impact of the policy on Y (1.35) and L (1.36) to be found in a straightforward fashion.

3. Fiscal policy the short run

To find the short-run impact of fiscal policy we use the goods and labour market equilibrium conditions (1.17) and (1.18), noting that in the short-run n is fixed, whilst l , y and π may all diverge from their long-run equilibrium values. Using the facts that:

$$\frac{dw}{dg} = (1 - \mu) f'' \frac{dl}{dg}; \quad \frac{d(\Pi/P)}{dg} = n(\mu f' - l(1 - \mu) f'') \frac{dl}{dg}$$

we have, upon differentiation of (1.17) and (1.18) with respect to g :

$$\begin{bmatrix} n f' (1 - \mu C_2) - (1 - \mu) f'' (C_1 - L C_2) & \frac{M^0}{P} C_2 \\ n (1 - \mu L_2^s f') - (1 - \mu) f'' (L_1^s - L L_2^s) & \frac{M^0}{P} L_2^s \end{bmatrix} \begin{bmatrix} \frac{dl}{dg} \\ \frac{dP}{dg} \end{bmatrix} = \begin{bmatrix} 1 - C_2 \\ -L_2^s \end{bmatrix} \quad (1.70)$$

(1.70) can then be solved for $dP/dg|_{SR}$ (1.32) and dl/dg . The latter then allows the short-run effects of the policy on Y (1.29), L (1.30) and w (1.31) to be found.

Notes

1. The implications of imperfect competition, often combined with institutional or behavioural rigidities (for example menu costs) for the conduct of government policy have been explored in a number of papers which fall within the New Keynesian School. The main emphasis of this literature has been on the question of monetary non-neutrality (for example, Ball and Romer (1990), Blanchard and Kiyotaki (1987), Caplin and Spulber (1987). For views on the relationship between New Keynesian and both New Classical and traditional Keynesian macroeconomics see Greenwald and Stiglitz (1987), Mankiw and Romer (1991) and Mankiw (1992).
2. In Dixon and Mankiw the utility function is Cobb-Douglas, whilst Startz adopts the slightly more general assumption of Stone-Geary preferences.
3. The crucial assumption in the DMS framework is constancy of marginal budget shares. This requires only that the Engel curves are linear, irrespective of whether they pass through the origin.
4. Thus income effects on labour supply are precluded.
5. See for example, Deaton and Muellbauer (1980).
6. Note further that in Dixon and Mankiw $\lambda = 0$.
7. See Blanchard and Kiyotaki (1987).
8. Note that the vertical axis in Figure 1.3 can be viewed as representing total output and employment, since both are proportional to n in long-run equilibrium.

9. Note that in the absence of full indexation of T then an increase in the money supply would result in a *more* than proportionate increase in the price level. The new long-run equilibrium would then be characterized by lower real money balances and a lower real value of taxation such that $(M_0 - T)/P$ remained unchanged.
10. See Appendix.
11. Note, from (1.25) that for $\mu = 0$, $dn/d\mu$ is strictly positive.
12. It is straightforward to confirm that monetary neutrality, a feature of the long-run as already discussed, extends to the short-run.
13. This decline, of course, also acts to offset directly the initial increase in labour supply.
14. Given the signs of the other terms in the expression, the condition $C_1 - LC_2 > 0$, referred to above, is sufficient to ensure $dw/dP|_{YY}$ is positive. For Cobb-Douglas preferences $C_1 - LC_2 = \alpha\beta(E + \frac{1}{w} \frac{M^0 - T + \Pi}{P})$.
15. We note that the short-run equilibrium values of aggregate employment and output, and the real wage are all decreasing in μ , whilst the relationship between π and the price level is in general ambiguous (though positive for Cobb-Douglas preferences).
16. Which describe long-run equilibrium; the argument extends in an obvious fashion to short-run equilibrium.
17. For a detailed analysis of taxation in the SMD framework, see Molana and Moutos (1991).
18. Referring to (1.32) note that $1 - C_2 + wL_2^s$ is unambiguously positive. Hence a sufficient condition for $dP/dg|_{SR} > 0$ is $L_1^s(1 - C_2) + L_2^s(C_1 - L) > 0$. With Cobb-Douglas preferences this latter expression becomes $(1 - \alpha - \beta) \frac{\beta}{w} [E + \frac{1}{w} \frac{M^0 - T + \Pi}{P}] > 0$. Hence for the Cobb-Douglas case the price level must rise.
19. For both Mankiw and Startz, money is absent from the utility function and, hence, $\beta = 1 - \alpha$.
20. With Cobb-Douglas preferences, $C_2 = \alpha$, $L_2^s = -\beta/w$.
21. Noting that for $\mu = 0$, $dw^m/d\mu = 0$, $dP/d\mu = 0$. (See (1.22) and (1.28)).
22. Since $dw^m/d\mu < 0$ for $\mu > 0$.
23. In the literature, the term social planner is used to refer to the first best given only the technology constraint. Even in the Walrasian literature, however, the constraint of a fixed number of firms is often imposed. To make matters clear, we use the term constrained social planner and specify explicitly the constraints we are imposing.
24. The term intertemporal PPF is a bit of a hybrid. The PPF is usually used to refer to a frontier that is purely defined by the technology (as in the case of the corresponding golden rule lines. However, the intertemporal PPF includes the intertemporal optimality condition, which incorporates the subjective discount rate.

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