Working Papers

TECHNOLOGICAL CHANGE, ENTRY AND STOCK MARKET DYNAMICS: AN ANALYSIS OF TRANSITION IN A MONOPOLISTIC ECONOMY

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CESifo Working Paper No. 641 (9)

January 2002

Category 9: Industrial Organisation

Presented at the CESifo Industrial Organisation Workshop, Venice, June 2001

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ISSN 1617-9595

* We would like to thank Ray Rees, Gianni Defraja, and members of the CESifo 2001 workshop in IO at Venice for their comments on an earlier draft. This paper was written whilst Huw Dixon was an ESRC Research Fellow. Faults remain ours.

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Abstract

This paper explores an equilibrium model for industry entry dynamics and technological change. We focus on the share valuation of firms in the transition as technology changes, and whether or not share prices are always increasing when technology improves. We find that there can be a *U*-shaped transition dynamic, so that an initial boom in share price is followed by a temporary fall in share price even though the underlying technology is improving.

JEL Classification: D4, O3, L1, D92.

Keywords: technology, share-price, entry, industry, dynamic.

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1 Introduction

In this theoretical paper we consider the share price dynamics induced by changes in technological progress - perhaps due to the introduction of a new technology or institutional reforms - and the resultant entry of firms. The focus is not so much in comparing the steady states before or after the change, but rather on the *transitional dynamics* of the industry or economy as it adjusts. The model is one with efficient markets and perfect foresight where fundamentals drive the stockmarket value.

Should we expect technological progress to lead only to increases in the level of the stockmarket (monotonic dynamics), *or should we expect non-monotonic behavior of boom followed by partial bust* (a $U-$shaped or overshooting dynamic)? Is it possible to have share values falling even when the underlying technology is improving?

We model a stylized monopolistic industry or economy in a continuous time general equilibrium setting with no uncertainty and perfect foresight. We adopt a model of entry found in Das and Das (1997) and Datta and Dixon (2000)[1], in which the cost of entry is increasing in the flow of entry (due to some congestion effect or other externality). The flow of entry is determined by an *intertemporal arbitrage condition* that equates the cost of entry with the present value of incumbency. This gives rise to a dynamic zero-profit condition: the present value of incumbents in each instant is equal to the cost of entry.

We first consider the case of a step increase in the *level* of technology with no other underlying growth. When an unanticipated improvement occurs, it causes a stock market boom: there is a jump in the share value, the current profitability of incumbents and an increase in the flow of entry. However, eventually entry drives the profit level back to zero and shares decline back to the initial value. There the initial boom is followed by a bust. In the case where the increase is anticipated, the share price increases prior to the improvement, and then decreases.

[1] See also Aloi and Dixon (2001) for an application to a macroeconomic framework.

We next consider the case of an economy with exponential technology growth. What happens when the pace of technological change unexpectedly increases in addition to a step improvement? There is an upwards jump in the stock market value of firms. This can *overshoot* the new balanced growth path, with the possibility that there will be a U-shaped dynamic: the initial boom is followed by a slump before tracking back to the higher growth rate. If the initial jump overshoots the new balanced growth path, there is a downward pull of share-prices towards the new balanced growth path which may dominate.

This stock market behavior reflects the behavior of entry: after an initial rush, there is a temporary slow down before eventually getting back on track. A similar pattern can occur if the change is anticipated. We believe that this might be an explanation of the behavior of the stockmarket in the late 1990s. The initial technological change causes a bonanza of profitable investment opportunities causing high profits for incumbents and lots of new firms to set up. The increase in entry reduces profitability and this may cause the flow of entry to reduce, if only in the short run. However, finally the long-run growth opportunities begin to come through and the economy gets onto the new higher growth path.

2 Entry, profits and Share Valuation.

We introduce the basic model of entry developed which presents the dynamic zero profit entry condition which in turn relates the flow of entry to current profitability, stock market valuation and changes in market valuation. There is a monopolistic industry with a continuum of n firms. Profits per firm π is taken to be a function of the number of firms n and a technology parameter α : $\pi = \pi(n, \alpha)$,where $\pi_\alpha > 0 > \pi_n$. We can define the zero-profit number of firms as an implicit function of α, $\pi(n^*, \alpha) = 0 : n^*(\alpha)$ with $dn^*/d\alpha > 0$.

The entry model here is as simple as possible. Following Datta and Dixon (2000) and Das and Das (1997)[2], we assume that at each instant t there is flow

[2] The assumption is also to be found in Ericson and Pakes (1995).

cost of entry[3] $q(t)$ which is assumed to be proportional to entry flow $\dot{n}$

$$q = \nu \dot{n} \quad \nu > 0 \tag{1}$$

Relationship (1) is based on the notion that there is a congestion effect: when more firms are being set up, the cost of setting up is higher. This might be because of a direct externality in the setting up of new firms, or due to the fixed supply of some factor involved in the creation of new firms (some specialized human capital or other input).

The flow of entry in each instant is determined by an *arbitrage condition*. There is some fixed return of r available elsewhere (this could be a government bond). The arbitrage condition requires the return on investing a dollar in setting up a new firm is equal to r:

$$\frac{\pi(n, \alpha)}{q} + \frac{\dot{q}}{q} = r \tag{2}$$

The LHS represents the return to investing a dollar in setting up a new firm. The first term is the price of a new firm (the number of firms per dollar $1/q$) times the flow operating profits the firm will make if it sets up: the second term reflects the change in the cost of entry. If $\dot{q}/q > 0$, then it means that the cost of entry is increasing which encourages earlier entry; $\dot{q}/q < 0$ implies entry is becoming cheaper, thus discouraging early entry.

We have two equations (1, 2): this is a two dimensional system $\{n, q\}$, where n is a state-variable and q a jump variable. We represent this as a second order differential equation in n:

$$\nu \ddot{n} - r\nu \dot{n} + \pi(n, \alpha) = 0 \tag{3}$$

If we know the explicit form of $\pi(n, \alpha)$, we can investigate solutions to (3) using numerical methods. Instead, we seek to find more general results with analytical solutions to the linearized system.

A crucial feature of the entry model is that the dynamic arbitrage equation implies that the cost of entry q equals the net present value (NPV) of an incumbent firm at each instant:

[3] Entry and exit are symmetric for simplicity, with $-q$ being the cost of exit.

Proposition 1 $q(t) = \int_{s=t}^{\infty} \pi(n(s), \alpha(s))e^{-r(s-t)}dt$

This proposition creates the crucial link between entry, technology and the stock market value of firms in the industry. In an efficient stock market the value of firm shares will be equal to q in each instant. We can trace the dynamics of share prices and how they respond to technological innovations through q. We thus have an *intertemporal zero-profit entry condition*: the expected profits of any entrant at anytime are zero. If this were not so, firms could revise the timing of their entry to coincide with entry when it was profitable.

3 Step change in the Level of technology.

First we analyze the case where α is a constant: $\alpha(t) = \bar{\alpha}$. There is a steady state at $n^*(\bar{\alpha})$, so we can linearize around this steady-state to obtain a linear non-homogeneous *SODE*

$$\ddot{n} - r\dot{n} + \frac{\pi_n}{\nu}n = \frac{\pi_n}{\nu}n^* \tag{4}$$

The eigenvalues are

$$\lambda_i = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \frac{\pi_n}{\nu}} \tag{5}$$

Clearly, since $\pi_n < 0$, one eigenvalue is stable ($\lambda < 0$), and one unstable ($\lambda^+ > 0$) so that the steady state is saddle-path stable.

In the infinite horizon case, we will want to rule out explosive paths, so we can define the solution in terms of the stable eigenvalue λ

$$\begin{aligned} n(t) &= n^* + [n_0 - n^*]\exp[\lambda t] \\ q(t) &= \nu\lambda[n_0 - n^*]\exp[\lambda t] \end{aligned} \tag{6}$$

The system (6) is depicted in *Fig.1* as a phase diagram in $\{q, n\}$ space. The $\dot{n} = 0$ is the horizontal line $q = 0$. The $\dot{q} = 0$ line by $q = \pi(n, \bar{\alpha})/r$, which is downward sloping since $\pi_n < 0$. The saddle path is the downward sloping line with arrows; unstable paths are depicted in grey. For a given initial position n_0, q jumps to the stable manifold. Thereafter $\{n, q\}$ evolve according to (6).

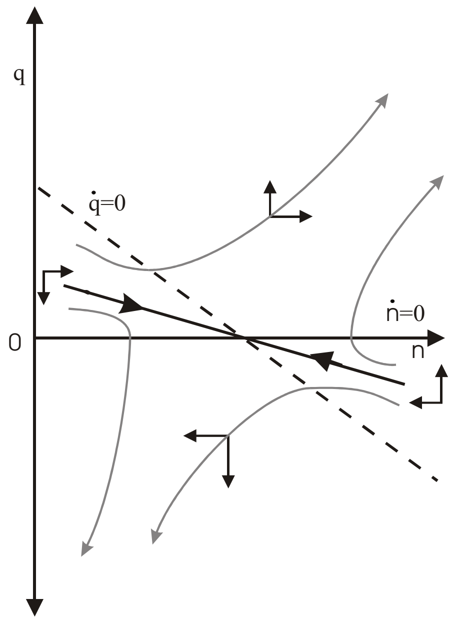

Figure 1: Phase digram in {n,q} space.

λ defines the *speed of convergence* to the steady state. When $\nu \simeq 0$, then there is a very small adjustment cost and the system converges rapidly to the steady-state (from 5, λ becomes large): when ν is very large, λ becomes close to zero and convergence is very slow. Hence the two cases of instantaneous entry ($\nu = 0$) and fixed n ($\nu = \infty$) are limiting cases of this entry process. Note that λ also defines the constant growth rate of q

$$\frac{\dot{q}}{q} = \lambda$$

Since $\dot{q}$ and q have opposite signs, q declines in absolute value to the steady state 0 at a constant rate along the saddle path. The flow of entry, $\dot{n} = q/\nu$, likewise declines to zero.

Note also that we can express q as a function of the current flow of profits, since the arbitrage condition can be written as $\pi = q(r - \lambda)$,

$$q = \frac{\pi(n)}{r - \lambda} \tag{7}$$

Note that the linearized solution reflects Proposition 1. In the linearized case we have the approximation for the flow of profits at time s as $\pi(n(s), \alpha) = \pi_n(n^*)(n(s) - n^*)$. From (6) $n(s) - n^* = [n(t) - n^*]e^{\lambda(s-t)}$, so that

$$\begin{aligned} NPV(t) &= \int_{s=t}^{\infty} \pi_n^*(n(s) - n^*)e^{(\lambda - r)(s-t)}ds \\ &= \frac{\pi(n(t))}{r - \lambda} \end{aligned}$$

Hence from (7), $q(t) = NPV(t)$ in the linearized system.

We will now look at a *permanent changes* which can either be *anticipated* or *unanticipated*. To make this concrete we will make α a shift variable that can either be α_1 or α_2 with $\alpha_1 < \alpha_2$, with corresponding free-entry (and steady-state) values $n_2^* > n_1^*$.

3.1 Permanent unanticipated increase.

First, the case of an unanticipated step increase in the level of technology. At $t = 0$ the system is in steady state with $n_0 = n_1^*$. There is a permanent increase at $t = T$ from α_1 to α_2. In this case, the $\dot{q} = 0$ line shifts rightward, and there is a new saddlepath passing through the new steady state n_2^*. The dynamics are depicted in Fig 2: q jumps to the new saddle path, from 0 to $q(T)$: there is a positive flow of new firms into the industry and the stock of firms increases towards the new equilibrium. The jump in q reflects the positive profits of the incumbents over the path to equilibrium: incumbents at $t = 0$ are able to earn positive profits throughout the adjustment path to the new equilibrium at n_2^*. Since the flow of profits falls over time, so does q.

3.2 Permanent anticipated Increase.

Now suppose that the permanent shift does not come out of the blue: the technological innovation occurs before it becomes available, or a policy is pre-announced with an explicit and credible timetable. The shift occurs at time T and is announced or becomes known at $t = 0$. For $t < T$, $\alpha = \alpha_1$; for $t \geq T$, $\alpha = \alpha_2$. Assume that the stock of firms at $t = 0$ is $n_0 = n_1^*$. The analysis here

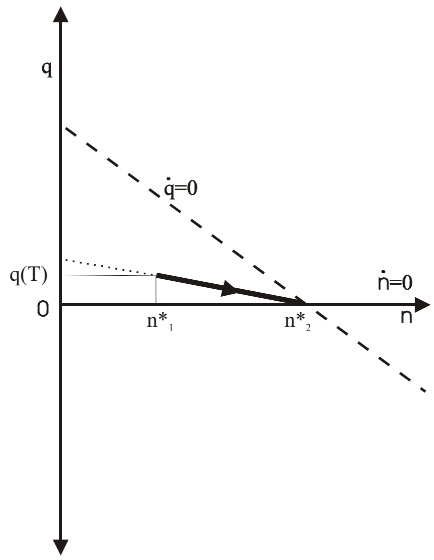

Figure 2: Unanticipated increase

divides into two periods[4]. In the second *post-shift* period the dynamics are given by the standard saddle-path around n_2^*. In the *pre-shift* period, $t < T$, the industry follows an *unstable* path relative to the current steady-state n_1^*. In period 0 the value of q jumps in anticipation of the increased profits earned along the path to the new steady state after T[5]. However, the shift does not occur until T: hence q jumps to a value *below* the prospective new saddlepath (below since profits will be less than if there was already the high value of α). $\{q, n\}$ then follow an unstable path diverging from the current n_1^* which joins up with the new saddlepath at point A in figure 3 at time T and thereafter converges to the new steady-state.

The initial jump $q(0)$ is larger (closer to the saddlepath) the smaller time T is: it joins the saddlepath at a smaller n the smaller T. Thus, for T close to 0, the industry jumps close to the saddlepath and joins it very soon at a value

[4] The dynamics at any instant t are governed by the current phase map and steady state (see Turnovsky (1997, pp.94-98).

[5] The *flow* of profits becomes negative until T because of entry.

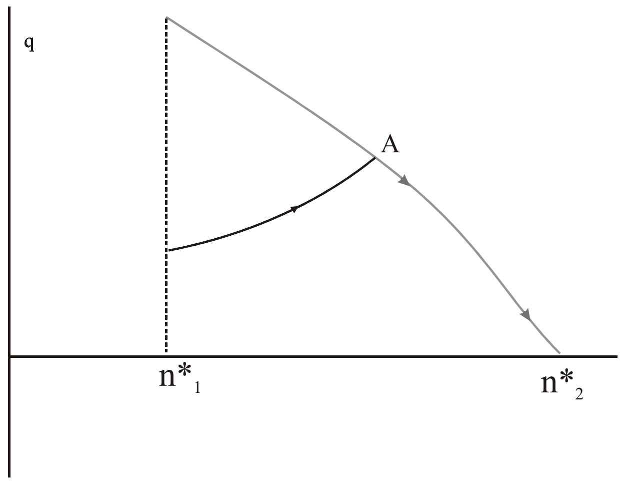

Figure 3: Permanent anticipated change.

of $n(T)$ close to $n_1^* = n(0)$. For T very large, $q(0)$ is close to 0 and joins the saddlepath with $n(T)$ close to n_2^*. This dynamics naturally reflects the fact that $q(t)$ is equal to the NPV of the firm at each instant.

The important point to note here is that share-price dynamics reflect both the technology and the entry process. Share prices overshoot the steady-state and bust follows boom as entry responds to technological advance.

4 Entry and technological growth.

Next we analyze the dynamics of entry and stock market valuation when α grows at an exponential rate $d : \alpha(t) = \alpha_0 e^{dt}$. For simplicity we adopt the linear functional form $\pi = \alpha - n$ so that $n^* = \alpha$, and $\pi_n = -1$: this can either be viewed as a linear approximation or as derived from an explicit model (see appendix A). As we show in appendix B and Datta and Dixon (2000), the analysis of this section holds for any $\pi(n, \alpha)$ which is homogeneous of degree 1 in $\{n, \alpha\}$.

The dynamic system can be written as a linear $SODE$ with a time-varying constant:

$$\ddot{n} - r\dot{n} - \frac{1}{\nu}n = -\frac{1}{\nu}\alpha_0 e^{dt} \tag{8}$$

The general solution to the homogeneous equation takes the standard form, with eigenvalues (5). A particular solution $\bar{n}(t)$ for the non-homogeneous equation is[6]

$$\bar{n}(t) = \left[\frac{1}{1 + \nu d\,(r-d)}\right]\alpha(t) \tag{9}$$

We assume that $r > d$, a sufficient condition for the NPV of profits to be defined. The particular solution grows at a constant rate d and involves strictly positive profits when $\nu d > 0$, since the number of firms is less than the zero-profit number. If $\nu = 0$, then we have the instantaneous entry case, with $\bar{n}(t) = n^*(t) = \alpha(t)$ at each instant. Along the particular solution, the share price is $\bar{q}(t) = \nu d\ \bar{n}(t)$. Note that an increase in the rate of growth of technological progress *always* causes the particular solution to the share price to increase.

Combining the general solution of the homogeneous linear $SODE$ with the particular solution yields the general solution to non-homogenous $SODE$: $n(t) = \bar{n}(t) + A_1 e^{\lambda t} + A_2 e^{\lambda^+ t}$. Ruling out explosive paths implies $A_2 = 0$, and the initial condition for $n(0) = \bar{n}(0)$ yields $A_1 = n_0 - \bar{n}(0)$:

$$\begin{aligned} n(t) &= \bar{n}(t) + [n_0 - \bar{n}(0)]\,e^{\lambda t} \\ q(t) &= \nu d\bar{n}(t) + \lambda\nu\,[n_0 - \bar{n}(0)]\,e^{\lambda t} \end{aligned} \tag{10}$$

There is a *balanced growth path* $\bar{n}(t)$ which grows exponentially. At the beginning, there is an initial deviation from the balanced growth path. The equilibrium solution involves the deviation from the balanced growth path diminishing over time. The speed of convergence is governed by the size of λ, which from (5) is determined by ν.

Let us consider what happens if at some time T there is a combination of an unanticipated but permanent increase in the rate of technological progress, from $d \geq 0$ to $d' > d$ and also a step jump in technology $\Delta\alpha_T \geq 0$. Up to time

[6] see Simmons 1991, pp. 99-106.

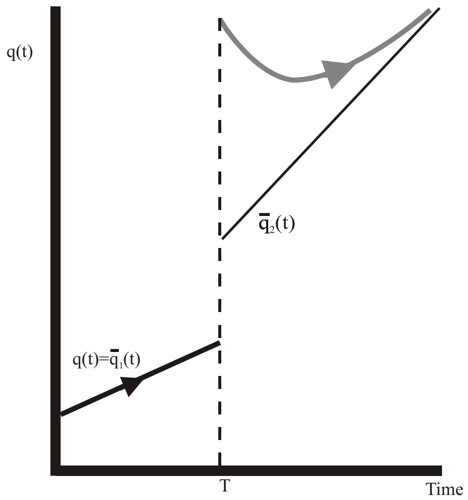

Figure 4: U-shaped share price dynamics.

T, we have $\alpha(t) = \alpha_0 e^{dt}$. At time T, α jumps from $\alpha_1 = \alpha_0 e^{dT}$ to $\alpha_2 > \alpha_1$. After T, $\alpha(t) = \alpha_2 e^{d'(t-T)}$. We define $\Delta\alpha_T = \alpha_1 - \alpha_2$.

The corresponding particular solutions are $\bar{n}_1(t)$ before T and $\bar{n}_2(t)$ after T. Prior to T the economy is following its balanced growth path: $n(t) = \bar{n}_1(t)$. The dynamics for $t \geq T$ are

$$\begin{aligned} n(t) &= \bar{n}_2(t) + [\bar{n}_1(T) - \bar{n}_2(T)]\, e^{\lambda(t-T)} \\ q(t) &= \nu d\bar{n}_2(t) + \lambda\nu\, [\bar{n}_1(T) - \bar{n}_2(T)]\, e^{\lambda(t-T)} \end{aligned}$$

Note that the share price jumps *upwards* at T: $q(T) > \underset{t \to T-}{Lim}\, q(t)$.

We now turn to the question of whether the change in the value of firms $q(t)$ can be non-monotonic after T, displaying a $U-$shaped dynamic. In particular immediately after T the rate of change in the share-price is given by[7]

$$\dot{q}(T) = \nu d'^2 \bar{n}_2(T) + \lambda^2 \nu\, [\bar{n}_1(T) - \bar{n}_2(T)] \tag{11}$$

An increase in the rate of technological growth means that the underlying growth is faster (reflected in the balanced growth path): this is captured in the first RHS

[7] $\dot{q}(T)$ is the RHS time derivative (the limit as t tends to T from above).

term of (11). However, there is also the pull towards the balanced growth path, reflected in the second RHS term: if this pull is downward, then it can outweigh the underlying trend.

Proposition 2 *$U-$shaped share-price dynamics. $d' > d \geq 0$.*

(a) Let $\Delta\alpha_T > 0$.

(i) If $d = 0$, there exists $\bar{d}$ such that for $d' < \bar{d}$, $\dot{q}(T) < 0$.

(ii) Let $d' < \bar{d}$. There exists $\hat{d} > 0$ such that for $d < \hat{d}$, $\dot{q}(T) < 0$.

(b) Let $\Delta\alpha_T = 0$. If $r(d' - d) + \left(d^2 - d'^2\right) \geq 0$, then $\dot{q}(T) > 0$.

Part (a) of Proposition 2 analyses the case where there is a step increase $\Delta\alpha_T > 0$. If we start from a situation of *no growth*, $d = 0$, then a small increase in growth to $d' < \bar{d}$ will lead to a $U-$shaped dynamic as in Figure 4. A corollary of this is (ii): given some small ex-post growth rate $d' < \bar{d}$, a small level of initial growth ($d > 0$) will also result in $U-$shaped dynamic. Taken together, the first two parts of the proposition mean that if the growth rates are not too large, then there will be a $U-$shaped dynamic when there is a step increase in technology at T. This makes sense, since high growth rates of underlying technology will tend to overpower any downward pull towards equilibrium. Part (b) gives a sufficient condition for there to be monotonic dynamics when $\Delta\alpha_T = 0$. Proposition 2 thus shows that both types of dynamic are possible in this setting. Note also, from (11), that when there is overshooting of the new balanced growth path (this happens whenever $\bar{n}_1(T) < \bar{n}_2(T)$), the rate of growth of share prices will be below the new balanced growth rate and possibly be quite small for a time resulting in a period of stagnation.

5 Conclusion.

We have developed a simple model relating share valuation to entry and technological progress. Even with efficient markets and perfect foresight, we find that there can be interesting transitional dynamics. We have focussed on situations where technological improvement can co-exist with declining share valuation, the possibility of a short run boom followed by a (possibly temporary) bust. We believe that this provides a new perspective on the phenomenon that acts as an alternative to notions of speculative behavior and behavioral models (Shiller 2000). Whilst our model is very stylized, it can form the basis for a more complicated model with uncertainty and other features. We leave this for future work.

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7 Appendix A: $\pi(n, \alpha) = \alpha - n$.

We provide a simple example to give the specific functional form $\pi(n, \alpha) = \alpha - n$. There is a continuum of possible markets $[0, \infty)$, with a given mass of consumers at each point who purchase only from that market. In each market j there is a demand given by $Y_j^d = \frac{a}{4}(1 - p_j)$, where a can be seen as a quality variable or something reflecting background growth in demand as a result of increased productivity in the economy. At any instant t, there are $n(t)$ active firms, so that markets $[0, n(t)]$ are active, and markets $(n(t), \infty)$ are non-active. In the active markets, there is a single firm which sets the price. In the non-active markets, we can think of the choke off price $\bar{p} = 1$ as prevailing.

The technology is such that each firm employs one unit of labor, and that unit of labor produces $a(t)$ units of output in quality adjusted terms. The marginal cost of the firm is 0 for output below this level ($Y_j < \alpha$), and infinite for larger outputs ($Y_j > \alpha$). All firms use the same resource: the total supply of labor is increasing in the real wage $L^s = w$. Labor demand $L^d = n$, so that market equilibrium is $w(t) = n(t)$. Profits per-firm are then $\pi = \frac{a}{4}(1 - p)p - w$.

Profit maximization requires that an active firm chooses price to maximize revenue $p^* = \frac{1}{2}$, resulting in the maximized profits $\bar{\pi}(w) = \alpha - w$. Substituting in the labor market equilibrium, this yields the profit function $\pi(\alpha, n) = \alpha - n$.

Note that the formulation for α we have adopted can also be interpreted as a demand variable. This is not a bad thing. Whilst a pure process innovation in a partial equilibrium setting might be correctly interpreted as separate from demand in the short-run, in general technological innovations have both a demand and supply side. A technological product innovation has a clear demand element; even process innovation will increase total factor productivity in a general equilibrium setting and hence increase demand.

8 Appendix B: Proofs.

8.1 Proposition 1.

From the arbitrage equation we have $rq - \dot{q} = \pi(n, \alpha)$. Hence

$$\begin{aligned} rqe^{-rs} - qe^{-rs} &= \pi(n, \alpha)e^{-rs} \\ \int_t^{\infty} (rq - \dot{q})\, e^{-r(s-t)} ds &= \int_t^{\infty} \pi(n, \alpha)e^{-r(s-t)} ds \\ q(t) &= \int_t^{\infty} \pi(n, \alpha)e^{-r(s-t)} ds \end{aligned}$$

8.2 Permanent unanticipated step change

The analysis of a permanent unanticipated change is straightforward: the initial position is with a stock of firms below the new steady state $(n(T) = n_1^* < n_2^*)$, and the share price and flow of firms jump up to the saddle-path and converge on the new steady state as in equation (6). For $t \geq T$ the dynamics are

$$\begin{aligned} n(t) &= n_2^* + [n_1^* - n_2^*] \exp[\lambda\,(t - T)] \\ q(t) &= \nu\lambda[n_1^* - n_2^*] \exp[\lambda\,(t - T)] \end{aligned}$$

8.3 Permanent anticipated step change

A permanent step change occurs in period T from a_1 to α_2: this is "announced" in period 0. The dynamics can be divided into two stages:

- Post announcement, pre-change: $t \in [0, T)$

$$\begin{aligned} n(t) &= n_1^* + A_1 e^{\lambda t} + A_2 e^{\lambda^+ t} \\ q(t) &= A_1 \nu\lambda e^{\lambda t} + A_2 \nu\lambda^+ e^{\lambda^+ t} \end{aligned}$$

- Post change.$t \in [T, \infty)$

$$\begin{aligned} n(t) &= n_2^* + A_1' e^{\lambda t} \\ q(t) &= A_1' \nu\lambda e^{\lambda t} \end{aligned}$$

If we assume that pre-announcement the economy is in steady state we have $n(0) = n_1^*$, so that $A_1 = -A_2$. Hence the continuity of n and q at time T imply that

$$\begin{aligned} n_1^* - n_2^* &= A_1' e^{\lambda T} - A_1 \left(e^{\lambda^+ T} - e^{\lambda T} \right) \\ 0 &= A_1' \lambda e^{\lambda T} + A_1 \left(\lambda^+ e^{\lambda^+ T} - \lambda e^{\lambda T} \right) \end{aligned}$$

This gives us two equations in two unknowns $\{A_1' \ , A_1\}$

$$\begin{bmatrix} n_1^* - n_2^* \\ 0 \end{bmatrix} = \begin{bmatrix} -\left(e^{\lambda T} - e^{\lambda^+ T} \right) & e^{\lambda T} \\ \lambda^+ e^{\lambda^+ T} - \lambda e^{\lambda T} & \lambda e^{\lambda T} \end{bmatrix} \begin{bmatrix} A_1 \\ A_1' \end{bmatrix}$$

the determinant is $\Delta = \left(\lambda - \lambda^+ \right) e^{T(\lambda^+ + \lambda)} < 0$. This yields

$$\begin{aligned} A_1 &= \frac{\left(n_1^* - n_2^* \right) \lambda e^{\lambda t}}{\Delta} < 0 \\ A_1' &= -\frac{\left(n_1^* - n_2^* \right) \left(\lambda^+ e^{\lambda^+ T} - \lambda e^{\lambda T} \right)}{\Delta} < 0 \end{aligned}$$

Since $A_1 < 0$, the coefficient on the dominant root λ^+ is strictly positive, so that in the pre-announcement period both the number of firms and the flow of entry increase. Once the change has occurred, the number of firms continues to increase, but since $A_1' < 0$ the flow of entry falls. The value of q at the time of announcement is

$$q(0) = -\lambda \nu \frac{\left(n_2^* - n_1^* \right)}{e^{T(\lambda^+ + \lambda)}} > 0$$

Note that this is strictly positive, is larger the nearer (smaller) T, and increasing in $\alpha_2 - \alpha_1$ (i.e. $(n_2^* - n_1^*)$).

8.4 Growth: Solution

To solve (8), we need to find a particular solution to the non-homogenous equation. To do this we use the method of undetermined coefficients. We posit the solution $n(t) = Ce^{dt}$, so that (8) becomes

$$\begin{aligned} \ddot{n} - r\dot{n} - \frac{1}{\nu} n &= \frac{\alpha_0}{\nu} e^{dt} \\ Ce^{dt}\left[d^2 - rd + \frac{1}{\nu}\right] &= \frac{\alpha_0}{\nu} e^{dt} \\ C &= \frac{1}{1 + \nu d(r-d)} \alpha_0 \end{aligned}$$

Hence we obtain the particular solution $\bar{n}(t)$ and hence $\bar{q}(t)$ and $\bar{\pi}(t)$.

$$\begin{aligned} \bar{n}(t) &= \left[\frac{1}{1 + \nu d(r-d)}\right] \alpha_0 e^{dt} \\ \bar{q}(t) &= \left[\frac{\nu d}{1 + \nu d(r-d)}\right] \alpha_0 e^{dt} \\ \bar{\pi}(t) &= \left[\frac{\nu d(r-d)}{1 + \nu d(r-d)}\right] \alpha_0 e^{dt} \end{aligned}$$

For the NPV of profits to be defined, we require $d < r$ (the usual condition that the rate of growth is less than the discount rate). This is sufficient for $\bar{n}(t) \leq n^*(t)$.

Note that *method* of solution is valid for any homogeneous of degree 1 function $\pi(n, \alpha)$, not just $\pi = \alpha - n$. If π_n is homogeneous of degree 0, we can linearize around a particular value of (n/α) so that π_n becomes a constant. In this case the dynamic system is

$$\begin{aligned} \ddot{n} - r\dot{n} + bn &= Be^{dt} \\ \text{where} \qquad B &= \frac{\pi_n}{\nu}\left[\frac{n}{\alpha}\right]^* \alpha_0. \end{aligned}$$

and $b = \frac{\pi_n}{\nu}$. This is a $SODE$ with a time-varying constant. Applying the method of undetermined coefficients gives us the particular solution $\bar{n}(t)$

$$\bar{n}(t) = \left[\frac{-\pi_n}{-\pi_n + \nu d(r-d)}\left[\frac{n}{\alpha}\right]^* \alpha_0\right] e^{dt}$$

8.5 Proposition 2

Define the difference in the particular solution at T as $\Delta\bar{n}_T = \bar{n}_1(T) - \bar{n}_2(T)$:

$$\Delta\bar{n}_T = \left[\nu\alpha_1 \frac{r(d'-d) + \left(d^2 - d'^2\right)}{\left(1 + \nu d\,(r-d)\right)\left(1 + \nu d'\,(r-d')\right)}\right] - \frac{\Delta\alpha_T}{1 + \nu d'\,(r-d')} \tag{12}$$

since $r > d' > d \geq 0$, it follows that $r(d' - d) + \left(d^2 - d'^2\right)$ can be positive (e.g. whenever $d = 0$) or negative (e.g whenever $d = r/2$). Furthermore, if $d = 0$ then

$$\underset{d' \to 0}{Lim} \Delta \bar{n}_T = -\Delta a_T \tag{13}$$

8.5.1 part (a)

(i) At T we have the RHS time derivative (11)

$$\dot{q}(T) = \nu d'^2 \bar{n}_2(T) + \lambda^2 \nu \Delta \bar{n}_T$$

From (13), if $d = 0$

$$\underset{d' \to 0}{Lim} \dot{q}(T) = -\lambda^2 \nu \Delta a_T < 0$$

(ii) This is a corollary of (i). If think of $\dot{q}(T)$ as a function of $\{d, d'\}$, we have some pair $\{0, d'\}$ such that $\dot{q}(T) < 0$. Hence, since $\dot{q}(T)$ varies continuously with d close to 0, the inequality will be maintained if d is small enough.

8.5.2 Part (b)

If $\Delta \alpha_T = 0$ and $r(d' - d) + \left(d^2 - d'^2\right) \geq 0$ then from (12) $\Delta \bar{n}_T > 0$, since only the positive term in square brackets remains. Hence from (11), $\dot{q}(T) > 0$.