Entry dynamics, demand shocks and induced productivity fluctuations.*

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Abstract

This paper analyses a small open economy Ramsey model with an endogenous labor supply and no capital. The number of firms is subject to adjustment costs, so that the entry dynamics is determined endogenously. We find that with imperfect competition, there is a first order effect of a demand shock on productivity which is absent in the Walrasian case. This is due to the presence of locally increasing returns around the zero-profit steady-state. We solve and analyze the dynamic model for permanent and temporary demand shocks.

Keywords: entry, adjustment costs, Ramsey.

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1 Introduction

This paper explores the relationship between changes in demand and productivity resulting from variations in capacity utilization[1]. Changes in capacity utilization result from the interaction of demand with an endogenous entry dynamic: we adopt the approach found in Das and Das (1997), Aloi and Dixon (2001) and Datta and Dixon (2002) in which the cost of entry depends on the flow of entry due to some congestion effect or externality. This results in the gradual adjustment of firms towards the zero-profit equilibrium. In the short-run output per firm and capacity utilization vary with aggregate output, whilst in the long-run the zero-profit condition determines steady state capacity utilization.

In an economy of monopolistic firms there is a direct relationship between the level of capacity utilization and productivity which is stronger when monopoly power is greater, as is found in the data (Ryan 2000). In this paper we assume that firms have $U-$shaped average cost curves, so that the monopolistic free-entry equilibrium output will be on the downward sloping portion of the AC and below the efficient operating level defined by the minimum AC output (this is just the standard Chamberlin-Robinson excess entry result). There is thus direct relationship between market power and the degree of monopoly power (see Basu and Fernald 1997): the higher the markup of price over marginal cost, the greater the degree of locally increasing returns[2]. If capacity utilization varies from the initial free-entry equilibrium due to a demand shock, there will be a first order effect of capacity utilization on average costs; since average cost is simply the dual of factor productivity, a fall in costs represents an increase in productivity. This effect is absent in a Walrasian setting with free entry: since firms are at optimum scale, the AC curve is locally flat ($P = MC = AC$), with no relationship between capacity utilization and productivity.

The relationship between imperfect competition and increasing returns to scale has been emphasized before[3], but the contribution of this paper is to provide an explicit dynamic entry model. Existing papers assume either that there is instantaneous free entry so that the actual or expected profit is driven to zero, or that the number of firms is constant (possibly fixed by some long-run zero profit condition as in Hornstein 1993). We explore the role of entry dynamics in a simple small open economy with a Ramsey consumer and eliminate all other sources of dynamics other than entry. We consider changes in demand which

[1] By *capacity utilization* we mean how output of firms compares to some standard reference level such as the free-entry equilibrium or the technically efficient scale of production. See Cassel (1937) and Klein (1960) and Nelson (1989) for the definition of capacity utilization.

[2] This can easily be seen. Since $p = AC$ from zero profits, it follows that

$$\frac{p}{MC} = \frac{AC}{MC}$$

where AC/MC is the inverse of the elasticity of cost with respect to output. With perfect competition this is unity with local constant returns; a larger p/MC means greater increasing returns.

[3] Hall 1986,1990, Rotemberg and Woodford 1995, Basu 1996, Devereux et al 1996, Ambler and Cardia 1998, Cook 2001, Coto and Dixon 2002.

can be either permanent or transitory, anticipated or unanticipated. The entry dynamics induces an endogenous productivity dynamic through variations in output per firm, capacity utilization. We model the demand shock as a balanced budget fiscal policy shock. Since there is no actual technology change, all variations in measured labor productivity (such as the Solow residual) are caused by changes in capacity utilization and are transitory. However, even short-run changes can have long-run effects, since the stock of foreign bonds may be permanently affected.

The basic insight of the paper is that the output of the economy or industry depends not only on the total level of input (labor in this paper, but also capital[4] and intermediates in general), but also the way that this is divided between firms. Here consider the simplest of a symmetric industry where all firms are identical[5]: the number of firms should be viewed as an additional quasi-input, representing the organization of the industry/economy. In the short-run, the number of firms can adjust only slowly, so that output changes only through changes in labor. At the firm level, this implies that output per firm varies. In the long-run, the number of firms can adjust alongside labor, so that output can change even if output per firm is constant. The key point is that we would expect a completely different relationship between output and employment in these two cases. This reinforces the applied work of Caves D *et al* (1981) and Bendt and Fuss (1986), which emphasized the need to focus on firm level data to understand productivity, not just industry or economy wide data.

The outline of the paper is as follows. First, in sections 2 and 3 we outline the optimization problem of the household and the firm, introducing the entry model in section 3.2. In section 4 we put these together into a dynamic general equilibrium model, exploring both the steady state and the linearized dynamics. In section 5 we use the model to analyze fiscal demand shocks.

2 The household

There is a small open economy, with a world capital market interest rate r equal to the discount rate ρ of the Ramsey household This is both a convenient simplification and a common assumption (see Turnovsky 1997, p.23-34). Utility satisfies standard assumptions and depends on aggregate consumption C and leisure $\ell = 1 - L$, where L is the labor supply.

Assumption 1 $U(C, 1 - L)$, is twice continuously differentiable and strictly concave with $U_C > 0 > U_{CC}$, $U_\ell > 0 > U_{\ell\ell}$ and $U_{C\ell} = 0$.

Leisure and consumption are normal goods, with U additively separable in C and l, $(U_{C\ell} = 0)$. The household earns income from three sources: supplying labor at wage w, receiving interest income from net foreign bonds rb and

[4] Brito and Dixon (2000) consider a closed economy model of entry with capital and labor.

[5] In a more general and realistic model, the distribution of firm sizes would also be crucial (see for example Hopenhayn 1992, Ericson and Pakes 1995, Basu and Fernald 1997 and Das and Das 1997)

receiving profit income Π. As is standard, the household treats profit income as a lump sum payment.

$$\max \quad \int_0^\infty U(C, 1-L)e^{-\rho t}dt$$

subject to

$$\dot{b} = rb + wl + \Pi - C - G \tag{1}$$

where we assume that the government finances its expenditure G by a lump sum tax equal to expenditure in each instant[6]. The solution[7] to the above is defined by the equations

$$\begin{aligned} U_C - \lambda &= 0 && \text{(2a)} \\ -U_\ell + \lambda w &= 0 && \text{(2b)} \\ \dot{\lambda} &= 0 && \text{(2c)} \end{aligned}$$

along with the Transversality condition (TVC)

$$\underset{t\to\infty}{Lim} \lambda b \exp[-rt] = 0 \tag{3}$$

The solution to the households problem is simple. Since $U_{C\ell} = 0$, we can write optimal consumption as a (decreasing) function of λ only

$$C = C(\underset{-}{\lambda})$$

The presence of international capital markets means that since $r = \rho$ the household can completely smooth its consumption ($\dot{\lambda} = 0$)[8]. λ is an index of the *level* of utility derived from consumption: a high λ means a low level of consumption and *vice versa*. The optimal supply of labor depends both on the real wage w and λ

$$L = L(\underset{+}{\lambda}, \underset{+}{w})$$

The combined assumptions of a perfect capital market and $U_{C\ell} = 0$ mean that the model is very simple: consumption is constant and the labor supply varies with the real wage. Hence the only dynamics in the model are going to be due to entry.

The aggregate consumption good C is assumed to be a CES subutility function. There is a continuum of possible products, $i \in [0, \infty)$. At instant

[6] This is for convenience and avoids the need for introducing government bonds. Since Ricardian equivalence holds, the timing of taxation does not matter.

[7] These conditions are the first conditions for the current value Hamiltonian $H = U(C, 1 - L) + \lambda[wL + \Pi - C]$.

[8] Note that if $r \neq \rho$ then no interior steady state exists. The trajectory of consumption will then be either be increasing ($r > \rho$) or decreasing ($r < \rho$) through time. Another possibility is that r itself is a function of time.

t, there is a range of products defined by $n(t) < \infty$, so that $i \in [0, n(t))$ are available and $i > n(t)$ are not produced.

$$C = n^{\frac{1}{1-\theta}} \left[\int_0^n c(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}, \quad \theta > 1$$

The demand for each available product i takes the constant elasticity form

$$c(i) = p(i)^{-\theta} C$$

3 Firms: Technology, entry and exit

There is a continuum of potential firms, each producing only one product. At time t, labor is used by active firms $[0, n(t)]$ to produce output according to the following technology

$$y_i = f(L_i) - F$$

where f is a twice-continuously differentiable function with $f' > 0 > f''$ and $F > 0$ is a fixed flow overhead in terms of output. The combination of diminishing marginal product of labor and fixed overhead results in a *U-shaped* average cost (AC) and increasing marginal cost (MC); this is compatible with both perfect (Walrasian) and imperfect competition.

The number of active firms is denoted n. Throughout we will be assuming that labor markets function perfectly so that labor is allocated equally across firms, so that $L_i = L/n$. The *aggregate production function* is Homogenous of degree 1 in n and L :

$$Y = \Phi(L, n) = n.f\left(\frac{L}{n}\right) - nF$$

Note that for all (L/n) the aggregate marginal product of labor equals the firm-level marginal product (this is because labor is allocated equally across firms)

$$\frac{\partial y_i}{\partial L_i} = \frac{\partial Y}{\partial L} = \Phi_L = f'\left(\frac{L}{n}\right)$$

Clearly, $\Phi_{LL} < 0$ since $f'' < 0$. The number of firms operating influences the level of output in the economy: it increases the level of overhead costs nF and decreases the level of employment per firm.

$$\begin{aligned}
\Phi_n &= f - f'\frac{L}{n} - F \\
\Phi_{nn} &= f''\frac{L^2}{n^3} < 0 \\
\Phi_{nL} &= -f''\frac{L}{n^2} > 0
\end{aligned}$$

The case of the *U-shaped* AC is significantly different from the case of $CRTS$

where $\Phi_{nn} = \Phi_{nL} = 0$, and the number of firms has no effect on aggregate output.

With our assumptions about technology, there is a clear optimal firm size. The technically efficient level of employment and output per firm are defined by the condition $\Phi_n = 0$

$$\Phi_n(L, n) = 0$$

Since Φ is homogeneous of degree 1, it follows that Φ_n is homogeneous of degree 0. Furthermore, $\Phi_{nn} < 0$, Φ_n is strictly monotonic in (L/n) and can be inverted[9], so that

$$\left(\frac{L}{n}\right)^e = \Phi_n^{-1}(0) \tag{4}$$

$$y^e = f\left(\left(\frac{L}{n}\right)^e\right) - F \tag{5}$$

The efficient number of firms conditional upon employment is

$$n = L.\left(\frac{L}{n}\right)^e$$

3.1 Profits

In this section, we determine the operating profits of an active firm, i.e. a firm that does not incur any entry costs. Due to imperfect competition, the firm maximizes profits given real wage w (using output price as the numeraire) by choosing employment to satisfy

$$w = (1-\mu)\Phi_L \tag{6}$$

Where μ is the Lerner index of monopoly[10]

$$\mu = \frac{p - MC}{P}$$

Since Φ is homogeneous of of degree 1 in $\{n, L\}$ we have

$$\Phi = L\Phi_L + n\Phi_n$$

Hence, given w from (6), the flow of operating profits at the firm and aggregate level are

$$\pi = \Phi_n + \mu\Phi_L\frac{L}{n} \tag{7}$$

[9] We define the function $\Phi_n\left(\left(\frac{L}{n}\right)^e, 1\right) = 0$, which can then be inverted in its first argument.

[10] In the macroeconomics literature, it is common to use the ratio p/MC as the markup: the two are directly related since

$$\frac{P}{MC} = \frac{1}{1-\mu}$$

$$n\pi = \Phi - wL = \Phi - (1-\mu)\Phi_L L \tag{8}$$

The zero operating profit condition when no entry cost incurred is $\pi = 0$, i.e.

$$\Phi_n = -\mu \Phi_L \frac{L}{n} \tag{9}$$

Since Φ is homogeneous of degree 1 in $\{n, L\}$, it follows that both Φ_n and Φ_L are homogeneous of degree 0 in $\{n, L\}$. It follows that both sides of (9) are homogeneous of degree 0 in $\{n, L\}$. Hence the free entry condition determines the *ratio* $(L/n)^*$, that is the level of employment and hence output per firm y^*.Comparing (4,9), in the Walrasian case ($\mu = 0$), the free-entry and zero-profit outcomes are the same. In both cases, firms are operating where $AC = MC$, at the bottom of the $U-$shaped AC curve. When $\mu > 0$ however, $\left(\frac{L}{n}\right)^* < \left(\frac{L}{n}\right)^e$,$y^* < y^e$: this is the standard *excess capacity* result of Chamberlin and Robinson. With monopolistic competition, free entry leads to excess entry and firms operate on the decreasing part of the AC curve (there are locally increasing returns to scale).

Under symmetry, aggregate free entry output of the consumption good is given by $\Phi = ny^*$, which can be written as a function of L

$$\Phi = L.\Phi\left(\left(\frac{L}{n}\right)^*, 1\right) \tag{10}$$

Since Φ is homogenous of degree 1 in (n, L), the fixing of the ratio (L/n) in (9) means that Φ becomes proportional to L. Free entry imposes long-run $CRTS$ on the relation between L and Φ irrespective of the technology at the firm level.

3.2 The Entry Decision

What determines the number of firms operating at each instant t? In this paper we employ the model developed from Das and Das (1996) by Dixon (2000) and Datta and Dixon (2002). At time t, there is a flow cost of entry $q(t)$ for each entrant (entry and exit are symmetric for simplicity, with $-q$ being the cost of exit at time t). The cost of entry is assumed to be increasing in the flow of entry $E = \dot{n}$

$$q = \nu E \tag{11}$$

The total entry costs incurred by entrants are therefore νE^2. The relationship between the flow of entry and the cost of entry is based on the notion that there is a congestion effect: when more firms are being set up, the cost of setting up is higher. We do not model this: however, this might be because of a direct externality in the production of new firms, or due to the fixed supply of some factor involved in the creation of new firms. In Dixon (2000) it is shown that this model can be derived form a locational setup where firms are situated along a real line representing location in some technological/product or geographical space. If the cost of entry depends on the distance of the new entrant from the nearest incumbent firm, then the same relation between entry flow and cost exists: a higher flow of entry means that firms more distant from $n(t)$ are setting up. Whilst exit and entry are treated symmetrically in this paper, this is not essential. It is possible to model exit differently (e.g. there is a fixed cost of exit, perhaps zero, as in Das and Das 1996 or Hopenhayn 1992), the alternatives being analysed in Dixon (2000).

The flow of entry in each instant is determined by an *arbitrage condition*. Suppose a firm is inactive: it can either set up in instant t or delay. The firm can either invest in setting up or not. The opportunity cost of funds is given by the return on the bond, r. This must equal the return on investing a dollar in setting up a new firm, given by the LHS of (12)

$$\frac{\pi}{q} + \frac{\dot{q}}{q} = r \tag{12}$$

where π is given by (7). The first LHS term is the number of firms per dollar $(1/q)$ times the flow operating profits the firm will make if it sets up: the second term reflects the change in the cost of entry. If $\dot{q}/q > 0$, then it means that the cost of entry is increasing, so that there is a capital gain associated with entry at time t; if $\dot{q}/q < 0$ it means entry is becoming cheaper, thus discouraging immediate entry. The arbitrage[11] condition equates the return on bonds with setting up a new firm, and is a differential equation in q, which determines the entry flow by (11).

With entry, the total profits are the operating profits of firms less the entry costs paid by the entrants

$$\Pi = n\pi - \nu E^2 = n\Phi_n + \mu\Phi_L L - \nu E^2 \tag{13}$$

In equilibrium, $q(t)$ represents the net present value of incumbency[12]: it is the present value of profits earned if you are an incumbent at time t. This arises since the entrants are indifferent between entering and staying out. When $q < 0$, the present value of profits is negative: in equilibrium this is equal to the cost of exit. In steady state, we have $E = q = 0$, so that the entry model implies the zero-profit condition. Entry costs are thus a disequilibrium phenomenon.

Note that our entry model has the standard models as limiting cases: when $\nu = 0$, we have instantaneous free entry so that (12) becomes $\pi = 0$ and there are zero profits each instant; if we have $\nu \to +\infty$, then changes in n become very costly and n moves little if at all which approximates the case of a fixed number of firms.

[11] The arbitrage equation can be written in a way directly analogous to the user cost of capital

$$\pi = q(r - \frac{\dot{q}}{q})$$

[12] See Datta and Dixon (2002) for proof.

4 The dynamic system

We are now ready to draw together the different elements of the economy in order to represent the economy as an integrated dynamic system. Since consumption is constant, we have three dynamic equations

$$\begin{aligned}
\dot{n} &= E = \frac{q}{\nu} \\
\dot{q} &= rq - \left[\Phi_n + \mu \Phi_L \frac{L}{n}\right] = Q(n, \lambda, q) \\
\dot{b} &= rb + wL + \Pi - C(\lambda) = B(b, n, \lambda, q)
\end{aligned}$$

where

$$\begin{aligned}
w &= (1-\mu)\Phi_L \\
\Pi &= n\Phi_n + \mu \Phi_L L - \nu \frac{E^2}{2}
\end{aligned}$$

The system above has a subsystem in $\{n, q\}$ which determines the dynamics of the whole system, the bond equation being a residual (see Turnovsky 1997). The bond equation along with the TVC condition, (3), then determines the equilibrium value of λ and corresponding b^*. We first specify the steady state conditional upon b^* (and hence λ^*) and then go on to use the linearized dynamics to derive b^*. As is common in open economy models of this type, the path to equilibrium influences the stock of bonds (there is hysteresis) through the balance of payments. Hence the steady-state cannot be fully analyzed independently of the dynamics of the system.

4.1 Steady-state

In steady state we have $\dot{n} = E = \dot{q} = \dot{b} = 0$. We begin by analyzing the steady state conditional on the steady state equilibrium value of bonds[13], b^*. In this case we have three equations in three unknowns $\{L, n, \lambda\}$

$$\Phi_n(L^*, n^*) = -\mu \Phi_L(L^*, n^*) \left(\frac{L}{n}\right)^* \tag{14a}$$

$$(1-\mu)\Phi_L(L^*, n^*) = \frac{U_\ell(1-L^*)}{U_C(C(\lambda^*))} \tag{14b}$$

$$C(\lambda^*) = w^* L(\lambda^*, n^*) + rb^* - G \tag{14c}$$

Equation (14a) means that there are zero-profits in steady-state. Since Φ is homogeneous of degree one in $\{L, n\}$ the free entry condition (14a) determines both the ratio $\left(\frac{L}{n}\right)^*$ and the wage w^*. Equation (14b) means that the wage

[13] To solve for the equilibrium value of steady state bonds we need to describe the dynamics of the economy, which we leave to the next session.

equates the MRS and the marginal product of labor. The final equation (14c) comes from the steady state condition for bonds: $B(b, n, l, q) = 0$.

We can represent the steady state in consumption leisure space. The Income Expansion Path (IEP) represents the consumption leisure choice given the equilibrium real wage w^*.

Fig. 1 here

The assumption of additive separability $U_{C\ell} = 0$ does not place any simple restriction on the shape of the IEP: for example it can be non-linear. Since both leisure and consumption are assumed normal, it is upward sloping. The steady-state budget constraint (14c) is linear with slope w and intercept[14] $rb^* - G$, the dynamics of the path to steady state being reflected in the divergence between steady state bonds b^* and initial bonds b_0. The steady state equilibrium is then the intersection of the IEP and the budget constraint at point A. Note that the zero profit condition (14a) determines the equilibrium real wage which, after substitution into the intratemporal efficiency condition (14b), allow us to derive a reduced form equation for L as a function of λ: $\mathcal{L}(\lambda) = L(\lambda, w^*)$. We can then determine the equilibrium level of λ^* by the output market clearing condition

$$G + C(\lambda^*) - w\mathcal{L}(\lambda^*) - rb^* = 0 \tag{15}$$

where (15) can be viewed as an excess demand function for the steady state in terms of the price of marginal utility λ. The first two terms of the expression above, representing the expenditure side, are decreasing in λ, while the income terms, $w\mathcal{L}(\lambda)+rb^*$, are increasing in λ; hence there exists a $\lambda^* > 0$ such that the economy is at the steady state equilibrium. We have now defined the steady-state for a given value of the steady-state bonds b^*: we now need to turn to the dynamics to derive the steady state stock of bonds.

4.2 Linearized system

Linearizing around the steady state we have

$$\begin{bmatrix} \dot{n} \\ \dot{q} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\nu\lambda^*} & 0 \\ Q_n & r & 0 \\ B_n & B_q & r \end{bmatrix} \begin{bmatrix} n - n^* \\ q - q^* \\ b - b^* \end{bmatrix} \tag{16}$$

where

$$\begin{aligned} Q_n &= \lambda^* \left[\Phi_{Ln} \frac{L}{n} (1 - \varepsilon_{Ln})(1 - \mu) + \mu L \Phi_L \frac{(1 - \varepsilon_{Ln})}{n^2} \right] > 0 \\ \varepsilon_{Ln} &\equiv L_n \frac{n}{L} \end{aligned}$$

Note that we use the following Lemma to restrict the range of ε_{Ln}

[14] As depicted, we have $rb^* - G > 0$. Of course, the intercept can be negative.

Lemma 1 $0 < \varepsilon_{Ln} < 1$

Proof. *See appendix A.* ■

$$\begin{aligned} B_n &= \Phi_n + \Phi_L L_n \\ &= \Phi_L L_n - \mu \Phi_L \frac{L}{n} \\ B_q &= -\frac{q}{\nu} \leq 0 \end{aligned}$$

Note that the effect of entry on bond accumulation is ambiguous when μ is large. On the one hand, there is the positive effect of n on labor supply and hence output: $\Phi_{nL} L_n > 0$ exactly as in the Walrasian case. On the other hand the effect of entry is to reduce profits: this offsets the effect and may reverse the sign. Clearly, for small μ the effect is positive as in the Walrasian case. The determinant of the sub-system in $\{n, q\}$ (16) is negative

$$\Delta = -\frac{Q_n}{\nu \lambda} < 0 \tag{17}$$

Let us denote the negative eigenvalue as Γ. The solution to the linearized system is

$$\begin{aligned} n(t) &= n^* + (n_0 - n^*) \exp[\Gamma t] && \text{(18a)} \\ q(t) &= (n_0 - n^*) \Gamma \nu \lambda \exp[\Gamma t] && \text{(18b)} \\ b(t) &= b^* + \frac{\Omega}{\Gamma - r} (n_0 - n^*) \exp[\Gamma t] && \text{(18c)} \end{aligned}$$

where

$$\Omega = B_n = \Phi_L L_n - \mu \Phi_L \frac{L}{n} \tag{19}$$

Note that the sign of Ω is ambiguous

$$sign\ \Omega = sign\ \left(\varepsilon^*_{Ln} - \mu\right)$$

in the Walrasian case ($\mu = 0$), $\Omega > 0$ and the accumulation of firms leads to a *reduction* in bonds. The main mechanism here is that there is a positive effect of n on labor supply and output ($\Phi_{Ln} > 0$), so that having too few firms means that wages, labor income and home production are below their steady state level. To maintain consumption, this low level of income is compensated by higher than steady state imports, financed by running down bonds[15]. However, if μ is large enough then bonds may increase as firms are accumulated. This is because the level of profits along the path to equilibrium is large: whilst the number

[15] It is important to understand that an *increase* in firms per se makes wages higher. However, the number of firms is *increasing* because it is *below* the steady-state. The stock of bonds decreases not because the accumulation of firms lowers income, but because entry implies that the initial level of n was low in the first place and remains below the steady-state along the dynamic path.

of firms is below equilibrium, the extra profits generated are enough to exceed the adjustment costs and lower wage. In addition, there is a capacity effect, so that productivity is higher whilst the number of firms is below equilibrium (for $\mu > 0$, free-entry leads to excessive number of firms in steady-state). In the limiting case of $CRTS$ ($f'' = 0$, $\varepsilon_{Ln} = 0$), the flow of entry leads to an increase in the stock of bonds: this is because an increase in n has no effect on wages and a negative effect profits, so that n below its steady state implies income above the steady state.

The phase diagram of the system in $\{n, q\}$ space is depicted in Figure 2. The downward sloping line represents the combinations of $\{n, q\}$ for which $\dot{q} = 0$ and the arbitrage condition is satisfied $\pi = qr$.

Fig. 2 here

Above the $\dot{q} = 0$ line, the arbitrage condition implies that $\dot{q} > 0$; below it implies $\dot{q} < 0$. The $\dot{n} = 0$ phase line corresponds to the $n-$axis, since $\dot{n} = 0$ whenever $q = 0$. The saddle-path is downward sloping between the horizontal axis and the arbitrage line.

The linearized dynamics gives an explicit solution for steady state bonds as a function of λ and the initial condition n_0.

$$b^* = b(\lambda^*) = b_0 - \frac{\Omega}{\Gamma - r}(n_0 - n(\lambda^*)) \tag{20}$$

where $sign\ b_\lambda = sign\ \Omega$. We can now rewrite the SS condition for bonds (14c) as a function of λ only

$$w\mathcal{L}(\lambda^*) + rb(\lambda^*) - C(\lambda^*) - G = 0 \tag{21}$$

Hence we now have three equations (14a,14b,21) to determine the three variables $\{\lambda, n, L\}$ in steady state.

The following is a useful result for what follows.

Lemma 2 $w\mathcal{L}_\lambda + rb_\lambda - C_\lambda > 0$

Proof. See appendix A. ■

Similarly to the case analyzed in the previous section, we can interpret the equation (21) as a steady state market clearing condition: the LHS is the excess of income over expenditure. The Lemma shows that this is strictly monotonic in λ. Hence, if a steady-state exists it is a *unique* steady state solution *for* λ^*. Existence follows from the Inada conditions on $U(C, 1-L)$ and the fact that b^* is bounded in (20) since n^* is bounded[16]. When λ is close to zero, L is very small and C is very large, with C unbounded as $\lambda \to 0$: hence there expenditure exceeds income; when λ is very large, C is very small and L is close to 1, so that there is an excess of income over expenditure. Hence for some intermediate value of λ (21) is satisfied.

[16] n^* is proportional to L^*, which lies in $[0, 1]$.

5 Fiscal demand shock

We will consider a demand shock in terms of a tax financed change in government expenditure. In order to properly understand this, we need to introduce a national income accounting framework. We define total consumption Y to consist of private and public consumption, and classify the expenditure incurred in setting up new firms as investment, $I = \nu E^2$.

- *Gross domestic product (GDP)*: $GDP = \Phi(L, n, A)$
- *Gross National product*: $GNP = \Phi + rb - \dot{b}$.
- *Total Consumption*: $Y = C + G$.

  These measures are clearly related: since we are considering a small open economy, all of these measures capture different aspects of the behavior of the economy. In steady state note that $Y = GNP = GDP + rb$.

5.1 Long run effects

Let us first look at the steady-state changes induced by a permanent and unanticipated change in G. Since in the long-run there is a zero-profit equilibrium, the basic properties of the long-run multipliers are not affected by the degree of imperfect competition.

Proposition 1 *Long-run multipliers for government expenditure.*
(a) $GNP \quad 1 > \frac{dY^*}{dG} > 0.$
(b) $GDP \quad 1 > \frac{d\Phi^*}{dG} > 0$, *and*
$sign \left[\frac{d\Phi^*}{dG} - \frac{dY^*}{dG} \right] = -sign\ \Omega$
(c) $\frac{dL^*}{dG} > 0;\ sign\ \frac{db^*}{dG} = -\ sign\ \Omega.$

Proof. See appendix A. ■

Fig. 3 here

The increase in government has two distinct effects on steady state consumption. First, there is the standard *resource withdrawal effect*: in Figure 3 this is represented by the vertical shift of the $LRBC$ by dG. Secondly, there is the *bond effect*: the increase in output and the number of firms causes a reduction in steady-state bonds if $\Omega < 0$ as depicted, since bond decumulation occurs along the path to the new steady state, represented by the further downward shift in the $LRBC$ by $r(db^*/dG)$. In the case of $\Omega > 0$ the bond effect will result in an outward shift in the $LRBC$. The overall reduction in leisure is given by the move from A to B: this is decomposed into the resource withdrawal effect A to A', and the bond effect A' to B. In terms of output, the decrease in private consumption is offset by the increase in government expenditure. The change in GNP which includes the bond effect is from A to B. The change in GDP excludes the bond effect, and is equivalent to the move from $A - A'$.

5.2 Impact Effects, capacity utilization and productivity

The instantaneous effects of an increase in G differ from the long-run effects since the number of firms is at its initial value.

Proposition 2 *Impact compared to long-run multipliers for $\mu > 0$.*

$$\text{(a)} \quad \frac{dY(\infty)}{dG} = \frac{dY(0)}{dG}$$

$$\text{(b)} \quad \frac{dL(\infty)}{dG} > \frac{dL(0)}{dG} > 0$$

$$\text{(c)} \quad \frac{d\Phi(0)}{dG} > 0, \frac{d\Phi(\infty)}{dG} > 0$$

$$sign\left[\frac{d\Phi(0)}{dG} - \frac{d\Phi(\infty)}{dG}\right] = sign\left[1 - \mu - \frac{dL(0)/dG}{dL(\infty)/dG}\right]$$

$$\text{(d)} \quad \frac{dw(0)}{dG} < \frac{dw(\infty)}{dG} = 0$$

Proof. *See appendix A.* ■

The increase in taxes G causes an increase in labor supply and this leads to a reduction in the wage (diminishing marginal productivity of labor). In the long run this is reversed as the economy moves to the free entry equilibrium and corresponding real wage w^*. Hence the initial increase in labor supply is smaller than the long run increase (b). The initial number of firms is below the new steady state. Firms are then accumulated which has the effect of stimulating the labor supply. Total consumption is constant *(a)*. GDP jumps initially in response to jump in the labor supply. In the long-run there are two effects at work: as firms are accumulated it changes the real wage and alters labor supply, whilst the additional firms reduce output due to the additional fixed costs. The second effect is inoperative when $\mu = 0$ and hence there are local $CRTS$, so that the labor supply effect dominates and the long-run By continuity, there exists $\bar{\mu} > 0$ such that the inequality is sustained: if there is a low level of imperfect competition the first effect will dominate and the long-run multiplier is bigger than the impact:

Proposition 3 *There exists $\bar{\mu} > 0$ such that for $\mu < \bar{\mu}$,*
$\frac{d\Phi(0)}{dG} < \frac{d\Phi(\infty)}{dG}$

Proof. *See appendix A.* ■

Lastly, we can examine the effect of an increase in demand on productivity, measured as the average product of labor $\mathcal{P} = \Phi/L$. Before the increase in G, the industry is in free entry equilibrium and operating at normal capacity $y = y^*$, with productivity[17] equals to the real wage, $\mathcal{P}^* = w^*$. The impact effect of an increase in demand is to increase output per firm.

[17] Note that in this paper we define productivity as output per unit labor ($\mathcal{P} = y/L$): since there is only one factor of production, $\mathcal{P}$ can be unambigously interpreted as productivity.

Proposition 4 *Productivity and capacity utilization.*

(a) If $\mu = 0$ *then* $\frac{d\mathcal{P}(0)}{dG} = \frac{d\mathcal{P}(\infty)}{dG} = 0$

(b) If $\mu > 0$ *then* $\frac{d\mathcal{P}(0)}{dG} > \frac{d\mathcal{P}(\infty)}{dG} = 0$

Proof. *See appendix A.* ■

When $\mu > 0$, the effect of an increase in government expenditure is to produce a transitory increase in the average product of labor, i.e productivity. The long run average product is equal to the steady state real wage which is of course unaffected by changes in government expenditure.

The behavior of productivity is best understood in terms of the change in *capacity utilization* caused by the change in employment. We can represent the technology in terms of the cost function as in Fig.4, with output on the horizontal axis and cost on the vertical axis: we depict the marginal and average cost functions on the assumption of diminishing marginal productivity of labor to yield the traditional U-shaped average cost function.

Fig 4 here

In Fig.4(a) we depict the equilibrium in the Walrasian case: the zero-profit long-run equilibrium output per firm is the technically efficient level y^e. Average cost equals marginal cost, so that average cost is flat, there being no first-order effects of capacity utilization on productivity (there is constant returns to scale in the neighborhood of the equilibrium). However, in Figure 4(b) we depict the long-run equilibrium when $\mu > 0$. In this case, we have the standard Robinson-Chamberlin excess capacity in long-run equilibrium. The AC curve is downward sloping, there being increasing returns in the neighborhood of the equilibrium. As output increases, capacity utilization increases towards the efficient level and there is a resultant increase in productivity.

The time path of the productivity following an unanticipated demand shock at time T is depicted in figure 5. There is an initial jump as capacity utilization increases, followed by a gradual decay back to the free-entry value as entry occurs. Clearly, productivity is endogenous in this model, determined by capacity utilization, which in turn is affected by the fiscal shock.

Fig. 5 here

The productivity dynamic induced by demand is endogenous. However, suppose that we mistakenly assume the productivity shock to be *exogenous*: in this case although there has been no underlying change in the technology parameter, we would infer that there had been a technology shock that decayed over time. We can relate this to the Solow growth residual, often advocated as a measure of technological change. From the definition of GDP :

$$\frac{d(GDP)}{GDP} = \left(\frac{\Phi_L L}{y}\right)\frac{dL}{L} + \left(\frac{\Phi_n n}{y}\right)\frac{dn}{n} + \frac{dA}{A}$$

To derive the true measure of technical progress dA/A, we substitute the actual equilibrium values from the model $\Phi_L = w/(1-\mu)$ and $\Phi_n n = \Pi - \frac{\mu}{1-\mu} wL - \nu E^2$.

Hence defining S^L as labor share of GDP and $S^{\Pi} = \left[\Pi - \nu E^2\right]/y$ as the share of dividends we have

$$\frac{dA}{A} = \frac{d(GDP)}{GDP} - \left(\frac{S^L}{1-\mu}\right)\frac{dL}{L} - \left(S^{\Pi} - \frac{\mu}{1-\mu}S^L\right)\frac{dn}{n}$$

The measured Solow residual (SR) is in this model

$$SR = \frac{d(GDP)}{GDP} - S^L\frac{dL}{L}$$

Hence, the difference between the true value of technological change and the measured SR is

$$SR - \frac{dA}{A} = \frac{\mu}{1-\mu}S^L\left[\frac{dL}{L} - \frac{dn}{n}\right] + S^{\Pi}\frac{dn}{n} \tag{22}$$

Assuming that we start from a free-entry equilibrium, so that $S^L = 1$, $S^{\Pi} = 0$, in the case of an unanticipated *demand* shock ($dA/A = 0$) on impact (since $dn/n = 0$), the Solow residual overestimates technological progress by

$$SR - \frac{dA}{A} = SR = \frac{\mu}{1-\mu}\frac{dL}{L}$$

That is, the conventional Solow residual will misinterpret the increase in productivity due to the capacity utilization effect as technological change. The size of this error is zero in the Walrasian case of $\mu = 0$, but is increasing in μ and proportional to the growth in employment when $\mu > 0$. *This is exactly as found by Ryan (2000), that the relationship between capacity utilization and productivity is stronger when there is more imperfect competition.* Over time, employment will fall and entry occurs, reducing the error, until in the long-run the growth in employment and the number of firms are equal (the term in square brackets in (22) becomes zero, as does S^{Π}).

5.3 Anticipated changes in government expenditure

In this section we will briefly examine the effect of an *anticipated* permanent change in government expenditure. The technical methodology follows Datta and Dixon (2001), so we will only illustrate the methodology here (see appendix B for an outline of the proof). First consider a permanent step increase in government expenditure that is to occur at time T but is announced at time $t = 0$. Let us assume that the economy at time 0 is in steady state prior to the announcement. The dynamics breaks up into two periods: from $0 < t < T$, and $t \geq T$. For each of these two phases there is the corresponding phase diagram, appropriate for the prevailing level of G. In the second phase, the economy will converge to the new steady state along the new saddle-path. As shown in Figure 6, in the initial phase after the announcement but before the change occurs, the economy will follow an unstable path.

Fig. 6 here

When the announcement is made, the present value of the incumbent firm q jumps. It jumps to a level below the new saddle-path, since the net present value would be higher if the increase occurred immediately. From this the economy follows and unstable path with q and n rising together as we get nearer the time of the increase. At time T the unstable path from the initial dynamics system joins on to the new saddle-path at point A: from here both q and n fall together towards the steady state. Note that in the initial phase, entry occurs despite the fact that profits are negative and becoming more so. At time T the level of profits jumps in response to the increase in expenditure (output and employment jump). The arbitrage equation is still satisfied at point T: the jump in π is exactly offset by the fall in $\dot{q}$ reflected in the kink at point q. The behavior of productivity and capacity utilization along this path is that until T, both productivity and capacity utilization decline below their steady state values. At T, there is a jump in both to above their steady state values, after which they both decline to the initial steady-state.

5.4 Temporary changes in government expenditure

Consider now a temporary *unanticipated* increase in G. The increase occurs at time $t = 0$ and continues until T, after which time the expenditure falls back to the new steady state[18]. The dynamics breaks up into two periods: from $0 < t < T$, and $t \geq T$. For each of these two phases there is the corresponding phase diagram, appropriate for the prevailing level of G. As shown in Figure 7, the new steady state will usually differ from the initial steady state $n_B \neq n_0$: although the shock is temporary, it has a permanent effect through its impact on the stock of bonds[19].

Fig. 7 here

When the announcement is made, the present value of the incumbent firm q jumps at time 0. The jump is to a level below the temporary saddlepath, since the increased profitability is only temporary. This leads to the economy following an unstable and non-monotonic path until at time T. Initially, there is a fall in q and increase in n: at point A,the increase in n peaks when $q = 0$. Then there is a period where both n and q are falling. The reason behind this is that although firms are profitable (the temporary shock is still present), the firms are anticipating the future decline in profitability. At time T the path joins up with saddlepath to the new steady state at point B: there are now too many firms. Again, at the point where there is an anticipated decline in expenditure, there are equal and opposite jumps in π and $\dot{q}$.

Fig. 8 here

[18] The formal analysis of this case closely follows Turnovsky 1977 (pages 94-98) and we refer the readers to his book.

[19] In Figure 7 we have $n_B > n_0$: whether the shock will result in an increase or decrease in steady state n will depend on the overall effect on L and hence the sign of Ω.

The time path of productivity is depicted in Figure 8. There is a jump at $t = 0$, after which it declines until point A (corresponding to A in Figure 7) is reached , after which this decline is partially reversed as firms exit in anticipation of the decline in demand, although still above its steady-state value. At time T (corresponding to B in Figure 7), when the increase in G is reversed, there is underutilization of capacity and productivity jumps to below $\mathcal{P}^*$ and gradually increases back to P^* as firms exit and the economy follows the saddlepath back to the steady-state.

6 Conclusion.

We have shown how the dynamics of entry is crucial to understanding the behavior of measured productivity in the response to demand shocks. Variations in output per firm, capacity utilization, has an important role in the short-run. We have developed this insight in an integrated and consistent manner to explore both the long-run and short run effects of changes in demand both when unanticipated and when anticipated, when permanent and when temporary. One interesting extension would be to make the markup endogenous and time varying. For example, we could develop the model to allow for oligopolistic markets, so that entry would be modelled with the number of firms as an integer which might be small as in Cook (2001). This would possibly raise many strategic issues such as entry deterrence and the optimal timing of entry by entrants, the latter which has yet to be modelled extensively at the microeconomic level. Another is to allow for capital (Brito and Dixon 2000 allow for capital and entry in a closed economy model). These tasks remain for future work.

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8 Appendix A: Proofs.

8.1 Proof of Lemma 1

Proof. Since Φ_L and Φ_n are homogeneous of degree zero, we have $\Phi_{Ln} = -\Phi_{LL}\frac{L}{n} = -\Phi_{nn}\frac{n}{L}$. Furthermore

$$L_n = \frac{-\Phi_{Ln}}{\Phi_{LL} + \frac{U_{ll}}{U_C}}$$

Hence

$$\Phi_{nn} + \Phi_{Ln}L_n = \frac{L}{n}\Phi_{Ln}(1 - \varepsilon_{Ln})$$

and

$$\varepsilon_{Ln} = \frac{\Phi_{LL}}{\Phi_{LL} + \frac{U_{ll}}{U_C}} \in (0, 1)$$

■

8.2 Proof of Lemma 2

Proof. Since $C_\lambda < 0$, it suffices to show that $w\mathcal{L}_\lambda + rb_\lambda > 0$.

$$w\mathcal{L}_\lambda + rb_\lambda = \Phi_L\mathcal{L}_\lambda\left[\frac{\Gamma(1-\mu) - r(1-\varepsilon_{Ln})}{\Gamma - r}\right] > 0$$

■

8.3 Proof of Proposition 1

Proof. From the steady state market clearing condition

$$G + C(\lambda^*) - w\mathcal{L}(\lambda^*) - rb(\lambda^*) = 0$$

hence from Lemma 2

$$\frac{d\lambda^*}{dG} = [w\mathcal{L}_\lambda + rb_\lambda - C_\lambda]^{-1} > 0$$

(a) The output multiplier for GNP is

$$\begin{aligned} Y^* &= G + C(\lambda^*) \\ \frac{dY^*}{dG} &= 1 + C_\lambda.\frac{d\lambda^*}{dG} \\ &= \frac{w\mathcal{L}_\lambda + rb_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} \in (0, 1) \end{aligned}$$

(b) For GDP the multiplier is

$$\begin{aligned}
\Phi &= \Phi\left(\mathcal{L}(\lambda), \left(\frac{n}{L}\right)\mathcal{L}(\lambda)\right) \\
\frac{d\Phi}{dG} &= \left[\Phi_L + \Phi_n \left(\frac{n}{L}\right)^*\right]\mathcal{L}_\lambda \frac{d\lambda^*}{dG} \\
&= \frac{w\mathcal{L}_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}
\end{aligned}$$

Hence the difference is

$$\frac{d\Phi^*}{dG} - \frac{dY^*}{dG} = \frac{-rb_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda} = -\frac{\Gamma\Omega}{\Gamma - r}\left(\frac{n}{L}\right)^* \frac{\mathcal{L}_\lambda}{w\mathcal{L}_\lambda + rb_\lambda - C_\lambda}$$

(c) and

$$\begin{aligned}
\frac{dL^*}{dG} &= \mathcal{L}_\lambda . \frac{d\lambda^*}{dG} > 0 \\
\frac{db^*}{dG} &= \frac{\Omega}{\Gamma - r}\left(\frac{n}{L}\right)^* \mathcal{L}_\lambda \frac{d\lambda^*}{dG}
\end{aligned}$$

■

8.4 Proof of Proposition 2

Proof. (a) Both C and λ jump to their new values $\{C^*, \lambda^*\}$, and hence Y too.

(b) The impact effect of fiscal policy is differentiated form the long-run effect by the fact that the number of firms is unchanged. Hence:

$$\begin{aligned}
\frac{dL(0)}{dG} &= L_\lambda(n,\lambda)\frac{d\lambda^*}{dG} > 0 \\
\frac{dL(\infty)}{dG} &= L_\lambda(n,\lambda)\frac{d\lambda^*}{dG} + L_n \frac{dn}{d\lambda}\frac{d\lambda^*}{dG} = \mathcal{L}_\lambda \frac{d\lambda^*}{dG} > 0 \\
sign\left[\frac{dL(\infty)}{dG} - \frac{dL(0)}{dG}\right] &= sign\left[L_n \frac{dn}{d\lambda}\frac{d\lambda^*}{dG}\right] \\
&= sign\ \varepsilon^*_{Ln}
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{d\Phi(0)}{dG} &= \Phi_L \frac{dL(0)}{dG} = \frac{w^*}{1-\mu}\frac{dL(0)}{dG} > 0 \\
\frac{d\Phi(\infty)}{dG} &= \Phi_L \frac{dL(\infty)}{dG} + \Phi_n \left(\frac{n}{L}\right)^* \frac{dL(\infty)}{dG} \\
&= w^* \frac{dL(\infty)}{dG}
\end{aligned}$$

$$\begin{aligned}
\frac{d\Phi(\infty)}{dG} - \frac{d\Phi(0)}{dG} &= w^* \frac{dL(\infty)}{dG}\left[1 - \frac{1}{1-\mu}\frac{dL(0)/dG}{dL(\infty)/dG}\right] \\
&= \frac{w^*}{1-\mu}\frac{dL(\infty)}{dG}\left[1 - \mu - \frac{dL(0)/dG}{dL(\infty)/dG}\right]
\end{aligned}$$

When $\gamma < 1$, the term in square brackets determines the sign of the RHS.

(d)

$$\Phi_{LL} L_{\lambda}(n, \lambda) \frac{d\lambda}{dG} = \frac{dw(0)}{dG} = sign\,[\Phi_{LL}]$$

■

8.5 Proof of Proposition 3

Proof. When $\mu = 0$,

$$\frac{d\Phi(\infty)}{dG} - \frac{d\Phi(0)}{dG} = w\frac{dL(\infty)}{dG}\left[\frac{dL(\infty)}{dG} - \frac{dL(0)}{dG}\right] > 0.$$

■

8.6 Proof of Proposition 4

Proof. From the definition of productivity, we have

$$\frac{d\mathcal{P}}{dL} = \frac{d(\Phi/L)}{dL} = \frac{L\Phi_L - \Phi}{L^2} = \frac{\mu\Phi_L}{L} = \frac{\mu}{1-\mu}\frac{w^*}{L}$$

Hence

$$\frac{d\mathcal{P}(0)}{dG} = \frac{d\mathcal{P}}{dL}\frac{dL}{d\lambda}\frac{d\lambda^*}{dG} = \frac{\mu}{(1-\mu)}\frac{w^*}{L}L_{\lambda}(n,\lambda)\frac{d\lambda^*}{dG}$$

This is strictly positive if $\mu > 0$, zero if $\mu = 0$. Since $\Phi(L, n)$ is homogeneous of degree 1 in (L, n) and the ratio L/n fixed by free entry, we have $\frac{d\mathcal{P}(\infty)}{dG} = 0$. ■

9 Appendix B: Analysis of anticipated changes.

The analysis here follows Datta and Dixon (2001) and uses standard techniques (e.g. Turnovsky 1997, pages 94-98). We will therefore just sketch the solution method taking the case of an anticipated step change, announced at time 0 to occur at time T. Once the change has occurred, the economy will follow the saddle-path to the new steady state. The initial (pre-announcement) stock of firms is in a steady state: we denote the initial steady state stock of firms as n_1. The eventual post-announcement steady state number of firms is n_2. Note that in this model, the eigenvalues depend on the steady state number of firms (**??**,**??**): the negative eigenvalues corresponding steady states are denoted Γ_i, $i = 1, 2$: the positive eigenvalue for the initial steady state is denoted Γ_1^+. Of course, the steady-state $q_i = 0$.

First, we describe the path over the initial phase over $t \in [0, T]$. When the announcement is made, before the actual increase in government expenditure occurs, the economy follows an unstable path relative to the initial equilibrium. Since the pre-announcement the economy is in equilibrium, $n(0) = n_1$, $b(0) = b_1$. Hence

$$
\begin{aligned}
n(t) &= n_1 + A_1 e^{\Gamma_1 t} + A_2 e^{\Gamma_1^+ t} && \text{(23a)}\\
q(t) &= A_1 \nu \Gamma_1 e^{\Gamma_1 t} + A_2 \nu \Gamma_1^+ e^{\Gamma_1^+ t} && \text{(23b)}\\
b(t) &= b_1 + \frac{\Omega_1}{\Gamma_1 - r} A_1 e^{\Gamma_1 t} + \frac{\Omega_1}{\Gamma_1^+ - r} A_2 e^{\Gamma_1^+ t} - \left[\frac{\Omega_1}{\Gamma_1 - r} A_1 + \frac{\Omega_1}{\Gamma_1^+ - r} A_2\right] e^{rt} && \text{(23c)}
\end{aligned}
$$

where Ω_1 is as in (19) evaluated at n_1. Note that setting $t = 0$, we have $A_1 = A_2$.

After the change in government expenditure occurs, $t \in [T, \infty)$, the economy follows a stable path to the new steady state

$$
\begin{aligned}
n(t) &= n_2 + A_1' e^{\Gamma_2 t} && \text{(24a)}\\
q(t) &= A_1' \nu \Gamma_2 e^{\Gamma_2 t} && \text{(24b)}\\
b(t) &= b_2 + \frac{\Omega_2}{\Gamma_2 - r} A_1' e^{\Gamma_2 t} && \text{(24c)}
\end{aligned}
$$

Since both $\{n, q\}$ are continuous at T, we have the two equalities in two unknowns $\{A_1, A_1'\}$.

$$
\begin{gathered}
A_1 \Gamma_1 e^{\Gamma_1 T} - A_1 \Gamma_1^+ e^{\Gamma_1^+ T} - A_1' \Gamma_2 e^{\Gamma_2 T} = 0\\
A_1 e^{\Gamma_1 T} - A_1 e^{\Gamma_1^+ T} - A_1' e^{\Gamma_2 T} = n_2 - n_1
\end{gathered}
$$

Now, simple substitution determines $\{A_1, A_1'\}$ conditional on n_2.

$$
\begin{aligned}
A_1' &= A_1 \left[\frac{\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}}{\Gamma_2 e^{\Gamma_2 T}}\right]\\
A_1 &= (n_2 - n_1) \left[e^{\Gamma_1 T} - e^{\Gamma_1^+ T} - \left(\frac{\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}}{\Gamma_2}\right)\right]^{-1}\\
&= (n_2 - n_1) \left[\frac{\Gamma_2}{\Gamma_2 \left(e^{\Gamma_1 T} - e^{\Gamma_1^+ T}\right) - \left(\Gamma_1 e^{\Gamma_1 T} - \Gamma_1^+ e^{\Gamma_1^+ T}\right)}\right]\\
&= (n_2 - n_1) \left[\frac{\Gamma_2}{e^{\Gamma_1 T} (\Gamma_2 - \Gamma_1) + e^{\Gamma_1^+ T} (\Gamma_1^+ - \Gamma_2)}\right] > 0
\end{aligned}
$$

Since $n_2 - n_1 > 0$ implies $\Gamma_2 - \Gamma_1 > 0$. Furthermore, both $\{A_1, A_1'\}$ can be seen as continuous functions of n_2 for $n_2 \geq n_1$.

The new steady state number of firms is determined by λ and comes from the dynamics for $b(t)$. Recall that we know b_1: hence from (23c), $b(T)$ is

$$
\begin{aligned}
b(T) &= b_1 + A_1 \left[\frac{\Omega_1}{\Gamma_1 - r} e^{\Gamma_1 T} - \frac{\Omega_1}{\Gamma_1^+ - r} e^{\Gamma_1^+ T}\right] && \text{(25)}\\
&\quad - \left[\frac{\Omega_1}{\Gamma_1 - r} - \frac{\Omega_1}{\Gamma_1^+ - r}\right] e^{rT}
\end{aligned}
$$

where on the RHS of (25) $\{\Omega_1, \Gamma_1, \Gamma_1^+\}$ are known, only A_1 needs to be determined. Turning to b_2, from (24c), we have

$$b_2 = b(T) + \frac{\Omega_2}{\Gamma_2 - r} A_1' \tag{26}$$

where $\{\Omega_2, \Gamma_2\}$ are functions of n_2 and hence b_2.

The two equations (25,26) give us a relationship between $\{A_1, A_1', n_2\}$ and b_2. We thus have an additional equation to determine n_2, since (14c) gives us the level of λ_2 given b_2, and hence n_2. In effect, we can conceive of the following algorithm: assume an arbitrary level of n_2: this then ties down $\{\Omega_2, \Gamma_2, A_1, A_1'\}$: we can then from (26) determine b_2 and hence λ. If the implied level of n_2 equals the initial value, then we have found the equilibrium value and the full solution to the model.

Does such a solution exist? First, note that in effect we have a mapping from n_2 onto itself. The constants $\{\Omega_2, \Gamma_2, A_1, A_1'\}$ are continuous functions of n_2; b_2 is a continuous function of these variables and the known values $\{b_1, \Omega_1, \Gamma_1, \Gamma_1^+\}$ from (26), and n_2 is a continuous function of b_2 from (14c). Second, note that n_2 belongs to a compact convex set: we have the lower bound n_1 and the upper bound $(L/n)^*$ (the number of firms in the economy when $L = 1$): $n_2 \in [n_1, (L/n)^*]$. Hence we have a continuous mapping of a compact convex set onto itself, which must posses a fixed point.

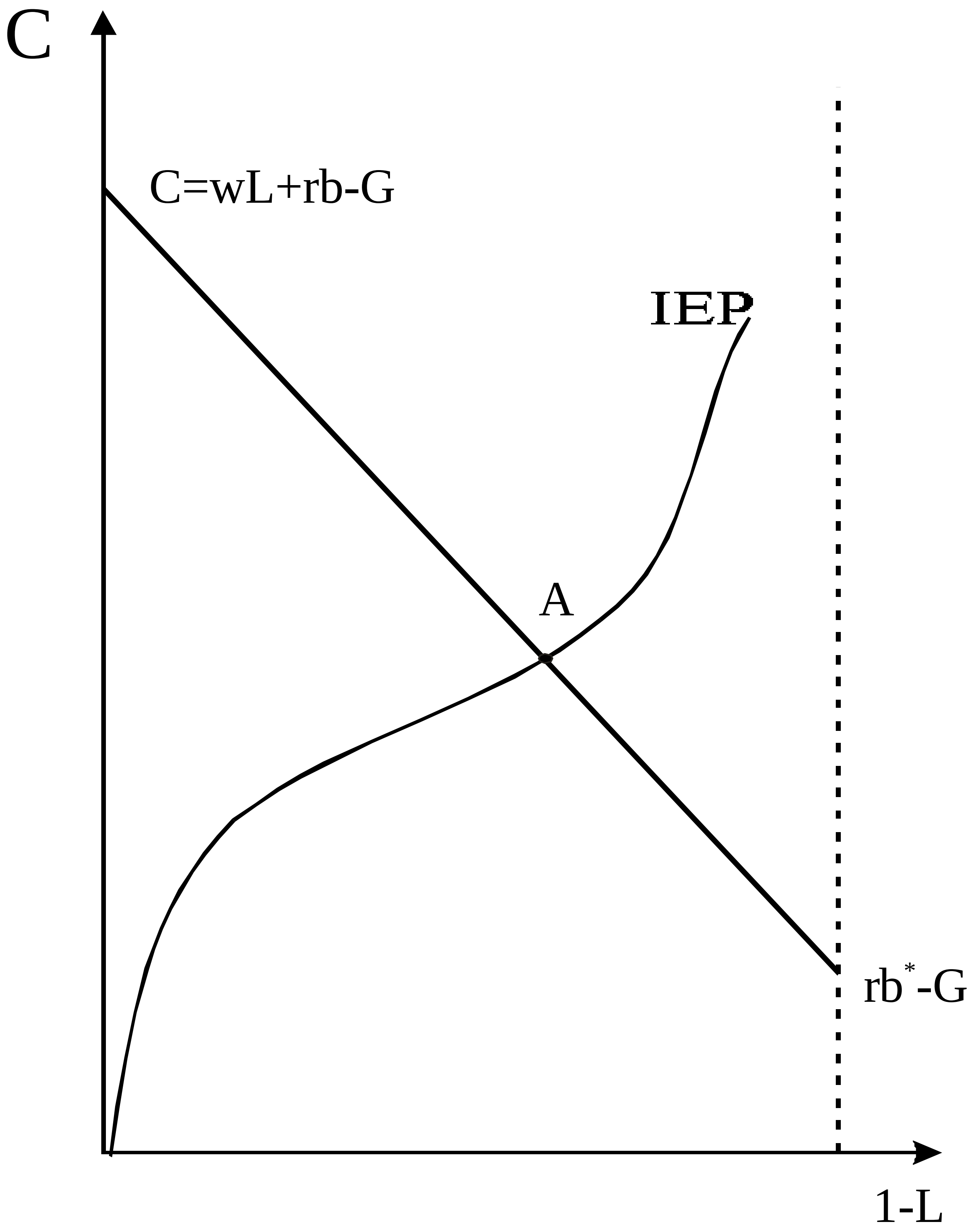

Figure 1: long run Equilibrium

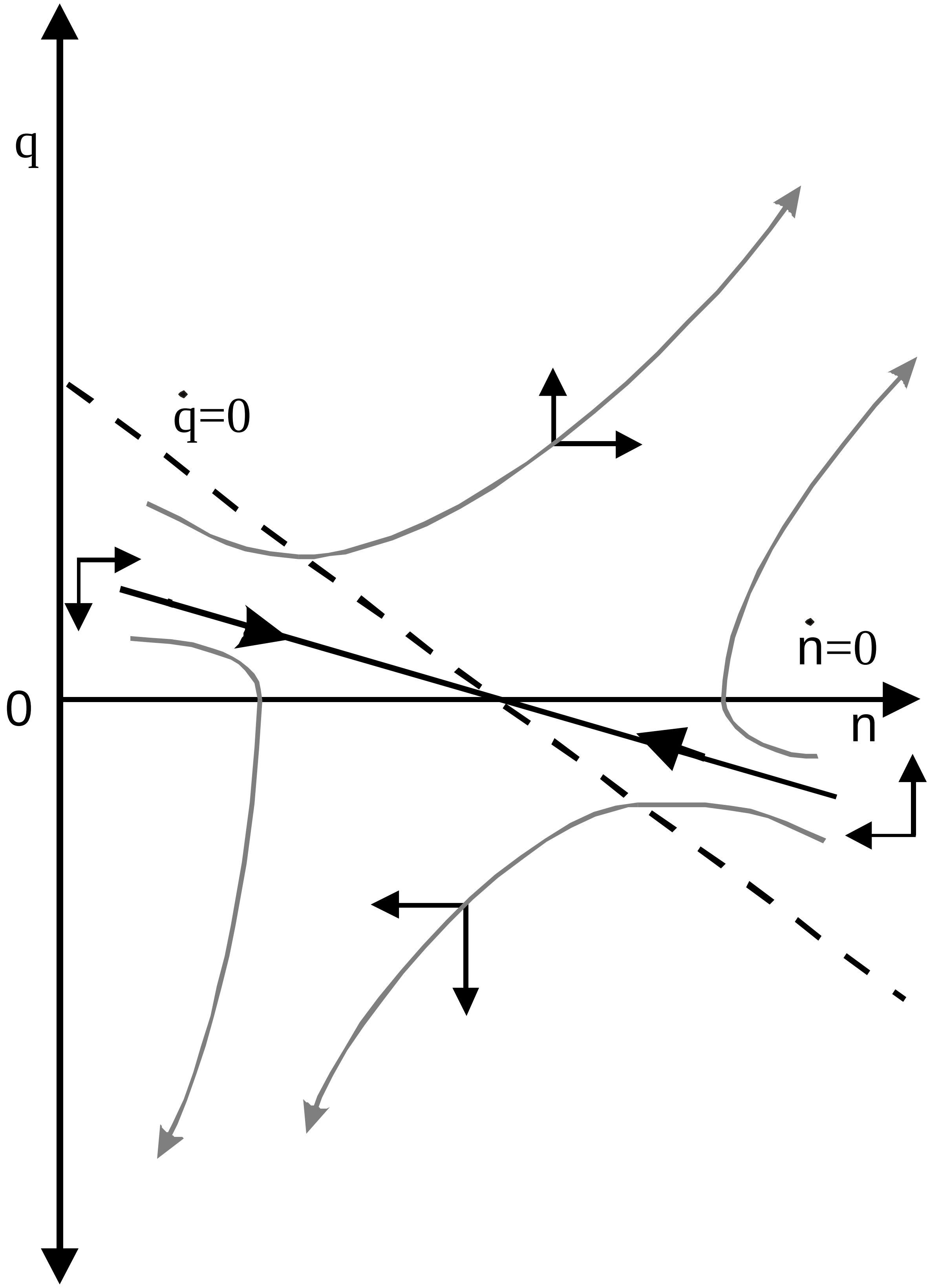

Fig.2: Phase Diagram in {n,q} space.

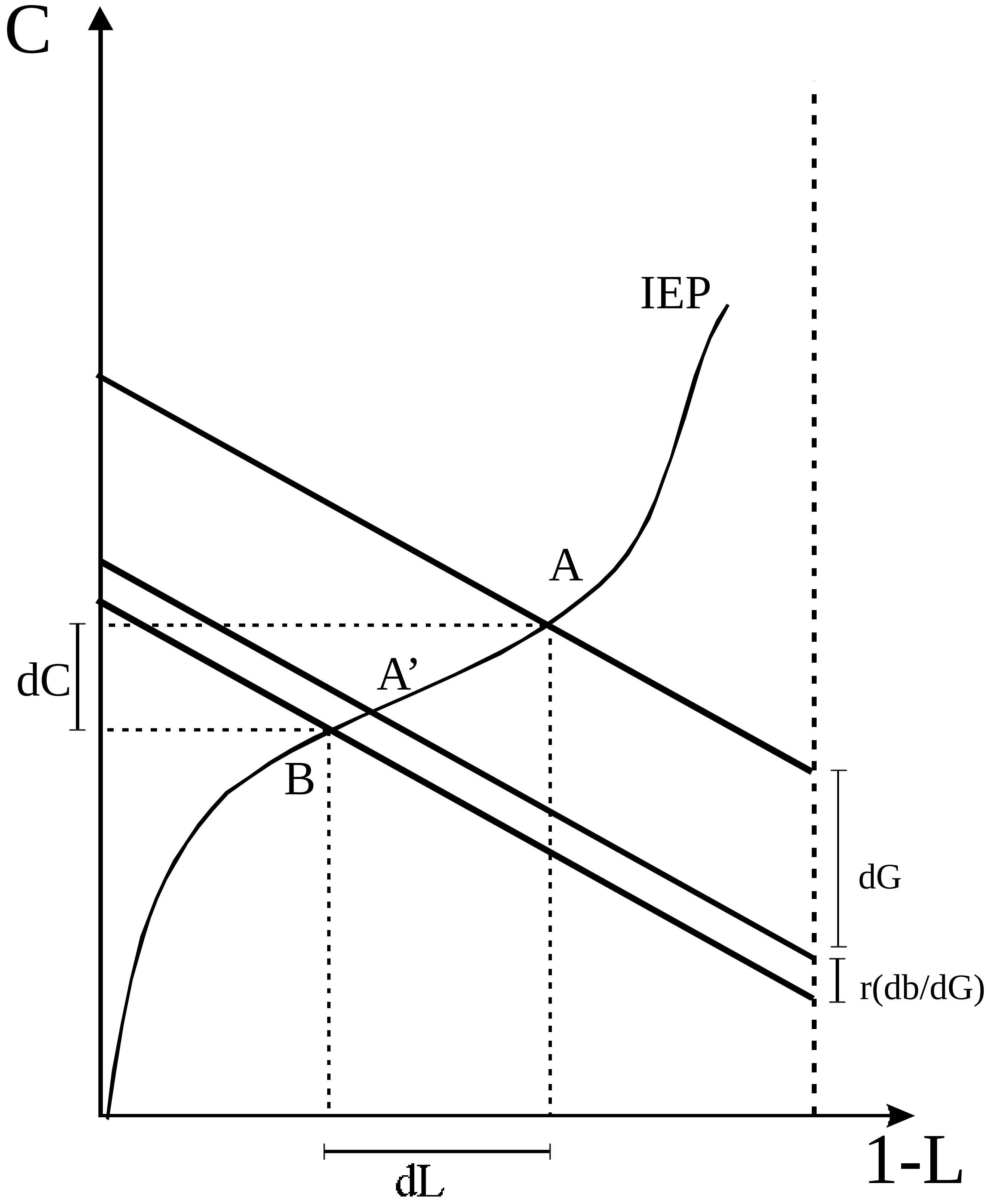

Figure 3: The Multiplier, Resource Withdrawal and Bond Effect.

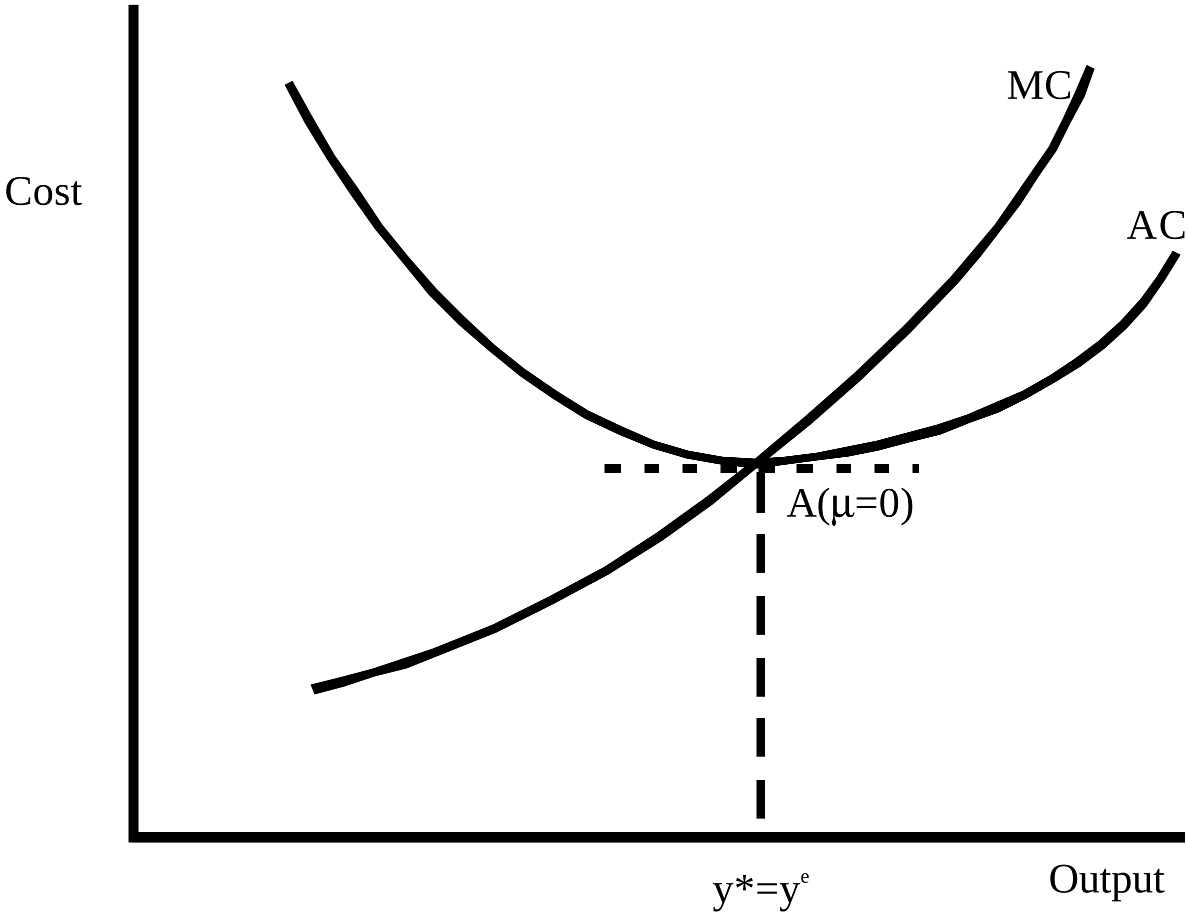

Fig 4(a) Capacity utilization and productivity when μ=0

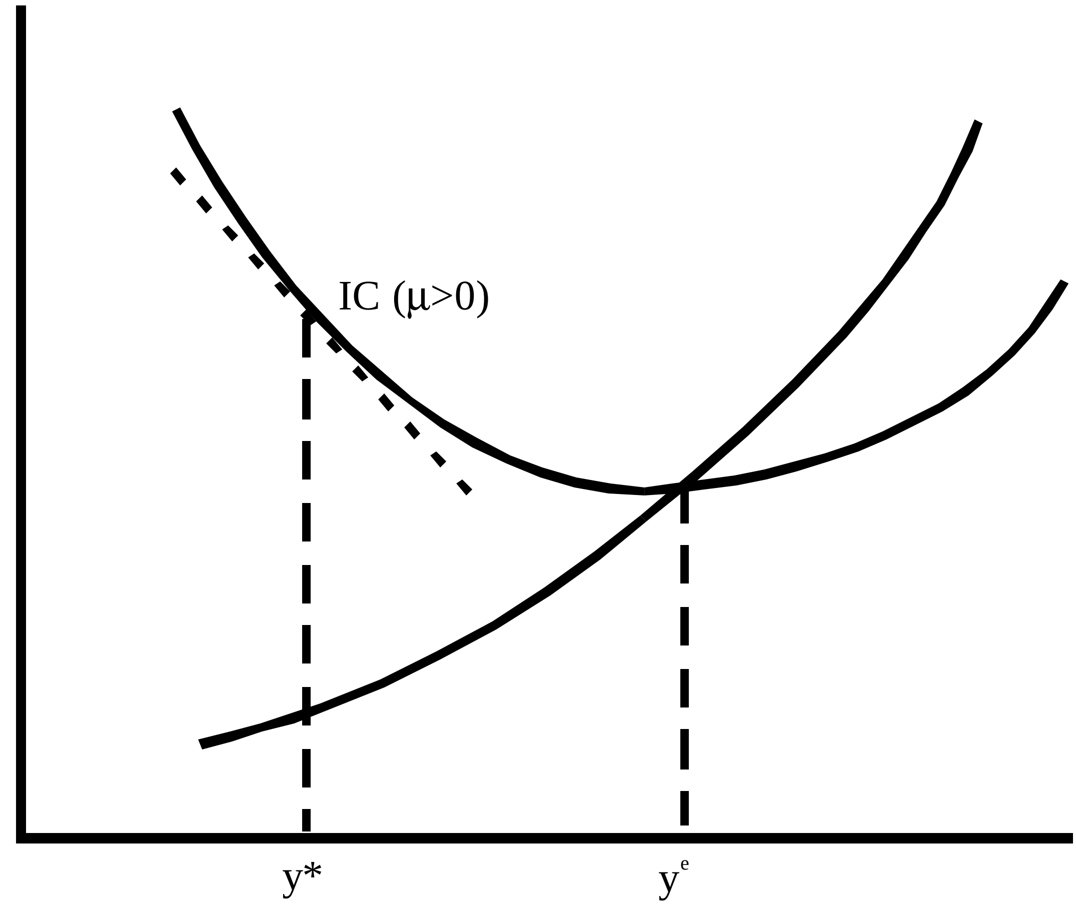

Fig 4(b) Capacity utilisation and productivity when μ>0

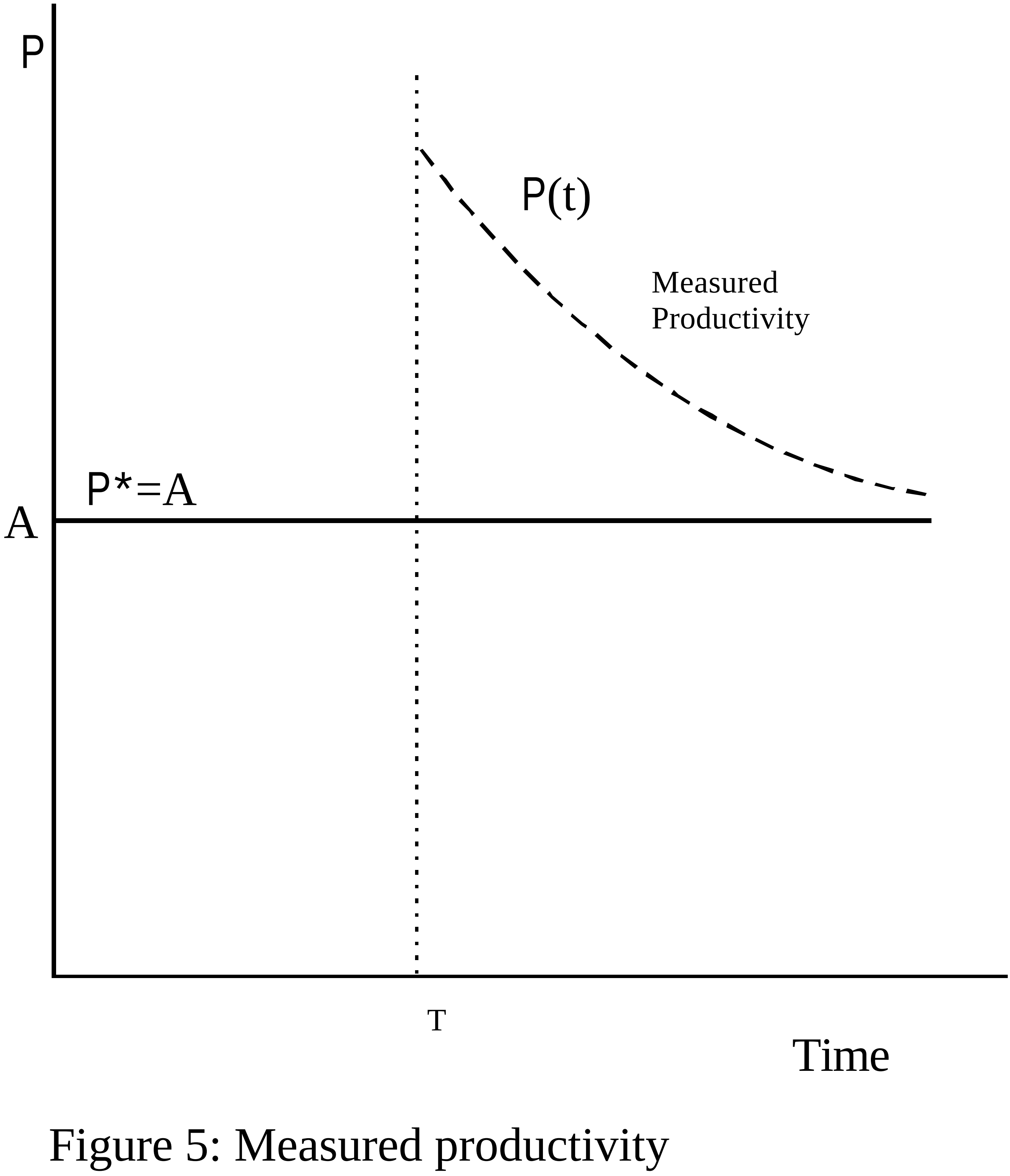

Figure 5: Measured productivity
After a demand shock at T.

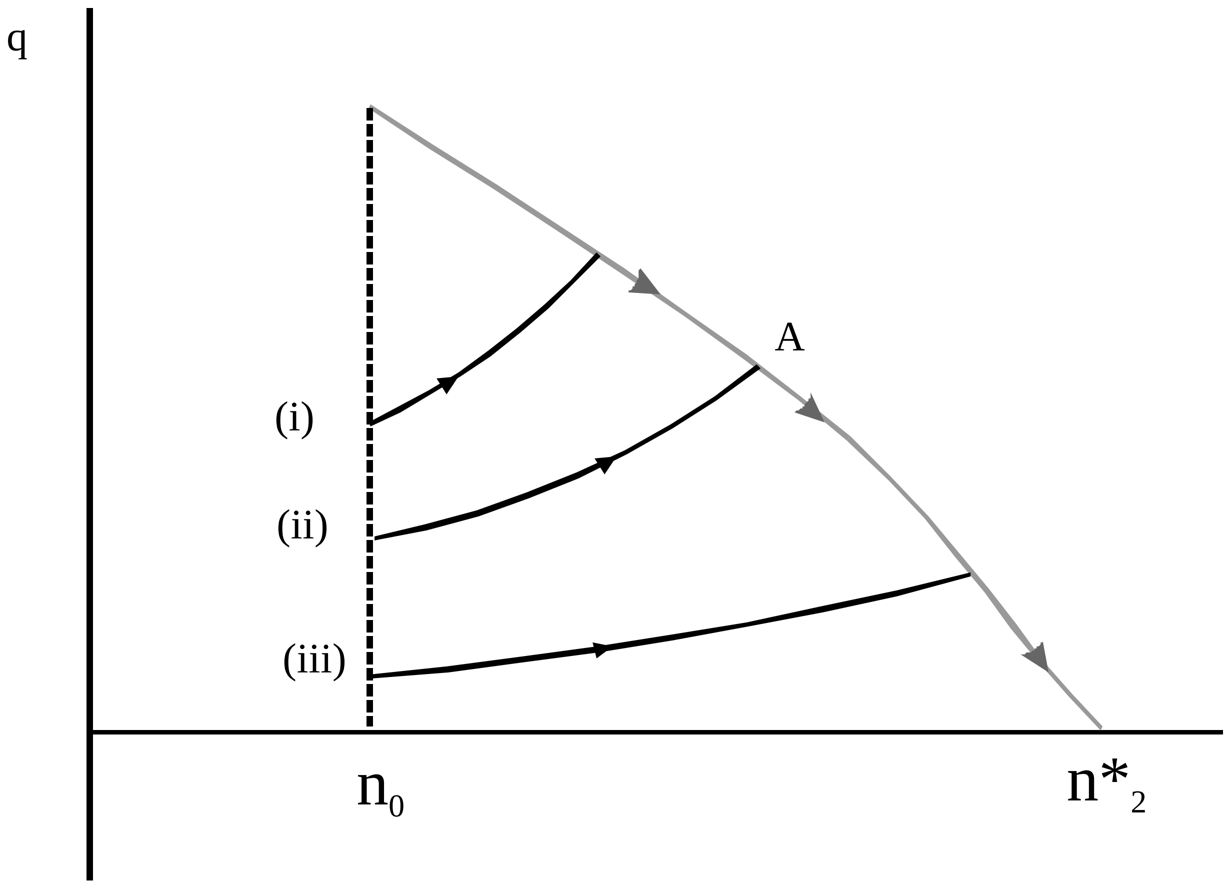

Figure 6: An anticipated Permanent Increase.

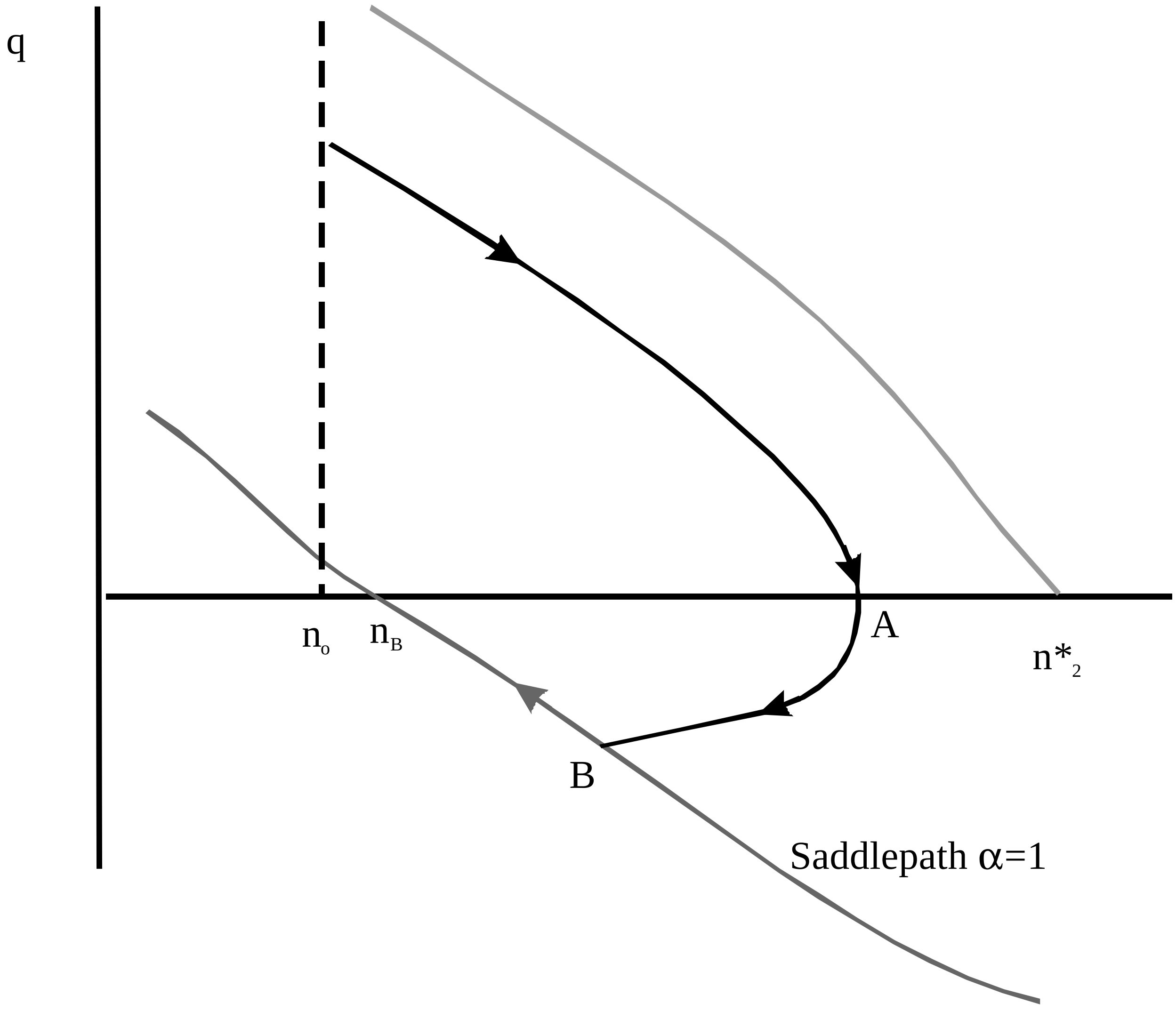

Figure 7: An unanticipated temporary change