A DSGE Model with Creative Destruction and Knowledge Spillovers

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Abstract

We develop a stylized two-sector DSGE model where the relative price of investment goods is not exogenously identified with sectoral TFP and varies endogenously according to the kind of aggregate shock hitting the economy. Our model incorporates endogenous firms entry/exit, thus generating creative destruction and knowledge spillovers in the K-sector as a permanent IST shock occurs. According to our model results are theoretically consistent with the permanent IST shock playing a relevant role for both growth and business cycle fluctuations. We interpret the IST shock as a sudden unexpected improvement of the new K-sector producers technology frontier, which then gradually spreads through incumbents. In addition, the endogeneity of the relative price of investment goods generates a feedback from the installation of investment goods into the stock of capital to the transformation of consumption goods into investment goods. This mechanism potentially invalidates the relevant contribution of the MEI shock as major business cycle driver as it generates a strongly prociclical relative price of investment goods which is not observed in the data.

1 Introduction

Productivity dynamics drives long run growth and plays a key role in determining business cycle fluctuations. Early stochastic growth models accounted for both aspects assuming neutral technological change. Motivated by the spectacular increase of capital intensity in the production of final goods over the last 50 years, more recent contributions investigate the distinct roles played by final goods- and investment specific- technological change (Greenwood et al., 1997 and Greenwood et al., 2000). In this respect, Fisher (2006), following a SVAR approach, finds that a permanent investment specific technology (IST henceforth) shock is the major source of both growth and business cycle fluctuations.

By contrast, Justiniano et al. (2011) (JPT hereafter) incorporate investment specific technology in an otherwise standard empirical DSGE model of the US. They draw a distinction between technological change that permanently affects the transformation of final into investment goods, identified with the relative price of investment, and temporary shocks that affect the production of installed capital from investment goods. They find that, given different sectoral productivity trends, the former shock drives the sectoral trend in relative productivity and bears no relevance at busines cycle frequency, whereas the latter is the most important driver of business cycle fluctuations.

These models postulate exogeneity both of innovation and of their diffusion, mimicked by the estimated peristence of shocks. What we do here is to build a DSGE model incorporating endogenous firm entry and exit in the capital sector, thus generating creative destruction and knowledge diffusion affecting aggregate variables. In particular, we bridge the gap between firms entry, knowledge spillovers and endogenous changes in the capital sector productivity affecting aggregate variables dynamics.

The transmission mechanism we have in mind is as follows. Investment goods producing firms (K-firms) are characterized by both idiosyncratic efficiency and decreasing returns to scale, and by a fixed production cost which is crucial to determine entry-exit conditions. In each period new entrants benefit from exogenous advances in the technology frontier, but entry and exit thresholds are affected by the endogenous relative price of investment goods. With a lag, the technology adopted by new entrants spreads to surviving incumbents. The role played by knowledge spillovers is also crucial for modeling K-firms exit endogenously without considering the idiosyncratic efficiency evolution. Threshold dynamics, i.e endogenous variations in the relative price of capital, determine the average efficiency of new entrants and therefore determine the process of technology diffusion.

To support intuition, we sketch here the implications of a permanent IST shock in our model. The shock is meant as an inflow of more productive firms in the market. This raises the supply of investment goods and lowers their relative price, thus increasing the productivity threshold which induces incumbents to exit the market. This, in turn, triggers a “creative destruction” event as only the least productive incumbents are wiped out. However, the subsequent knowledge spillovers from new entrants to incumbents further raise efficiency and lower the relative price of investment goods. Most importantly, and in contrast with the canonical formulation of the two-sector neoclassical model, we do not constrain the relative price of investment goods to be exogenously identified with the sectoral TFP growth, but we emphasize its effect on entry-exit thresholds. Moreover, unlike JPT, we do not need to postulate different BGP growth rates of sectoral productivitiy for our model to replicate the observed capital deepening process. In fact, even a relatively small shock can generate large variations the capital intensity in the final goods sector and in the relative price of investmet goods. Numerical simulations show that transition to the new steady state is very persistent.

To give an extent of the strength of our transmission mechanism, we find that when a temporary shock affecting the installation of gross investment into the stock of capital occurs, there is an endogenous feedback to the transformation of consumption- into investment goods. This endogenous feed back is such that it generates a demand shock in the K-sector rendering a strongly procyclical dynamics of the relative price of investment goods which is not observed in the data and this is in stark contrast with the permanent marginal efficiency of investment (MEI) shock playing a major role as business cycle driver as argued by JPT.

Our DSGE model incorporates nominal rigidities in the final goods sector, so that we can evaluate the supply-side effects of demand shocks which affect the relative price of investment-goods and therefore impact on entry-exit thresholds.

Our characterization of the K-sector endogenous evolution is loosely based on Asturias et al. (2017) who, based on the seminal work of Hopenhayn (1992), develop a growth model where firms entry and exit affect productivity through competitive pressures in the economy but they neglect the endogenous technology diffusion process modeled here. Clementi and Palazzo (2016) investigate the role that entry and exit dynamics play in the propagation of aggregate shocks, but neglect the role of sectoral productivity dynamics. We contribute to a rapidly expanding literature on endogenous entry and exit in DSGE models based on Bilbiie et al. (2012) (see also Etro and Colciago, 2010; Colciago and Rossi, 2015; Devereux et al., 1996; Chatterjee and Cooper, 1993; Jaimovich and Floetotto, 2008). They focus on business cycle fluctuations whereas we emphasize the interaction between technological change, technology diffusion and fluctuations at business cycle frequencies. In this regard our work is inspired by Sims (2011) and Canova (2014) who emphasize the importance of jointly considering the roles of the persistent but transitory productivity shocks of the RBC-DSGE literature and of the permanent shocks identified in the VAR literature (Galí, 1999).

We also contribute to the literature on technology diffusion. Indeed, another mechanism appointed by the related literature as a key aspect featuring economic growth is knowledge spreading. Parente and Prescott (1994) build a model of barriers to technology adoption able to explain per capita income disparity across countries. Among the others, Comin and Hobijn (2010) develop a neoclassical growth model of technology diffusion aiming to explain TFP differences at the country level. Differently, Comin et al. (2009) investigate the role of technology diffusion as business cycle driver. More recently instead, Anzoategui et al. (2016) focus on the endogenous cyclicality of technology diffusion as to explain the slowdown in productivity following the great recession.

The paper is organized as follows. Sections 2 describes the model economy. Section 3 is devoted to the interpretation of our results. Section 4 concludes. Technical details are left to the Appendix.

2 The model economy

In our economy there are 5 agent types. They are households, intermediate goods producers (or F-firms), retailers, K-firms and a central banker. Retailers are conceived to introduce price stickiness in the model and the central banker is in charge of conducting monetary policy.

They key players in the economy are K- and F-firms, respectively producing investment and final goods. The sequence of events is as follows. At time t, F-firms exploit factor services to produce goods which are sold to households. Households consume, save, supply labor and capital services. At the end of period t, K-firms are endowed with household savings, in the form of final goods, to produce investment goods which are then sold back to households who bear some adjustment costs in aggregating the new stock of capital.

K-firms are endowed with a decreasing return to scale technology, are characterized by idiosyncratic efficiency levels and face both variable and fixed production costs. The K-sector is characterized by entry and exit flows of firms. *NE*s draw their idiosyncratic efficiency levels from a new, more productive technology distribution. The combination of idiosyncratic productivity, technological change and fixed costs determines firms entry and exit flows in the K-sector. Our analysis emphasizes the distinct roles played by New Entrants (*NE*s) and Incumbents (*INC*s) in the K-sector.

Final goods producers are characterized by a permanent labor augmenting technology shifter also governing K-firms fixed cost dynamics. This will allow to investigate the distinct roles played by capital augmenting and by neutral technological change.

The sequence of events is summarized in the diagram below.

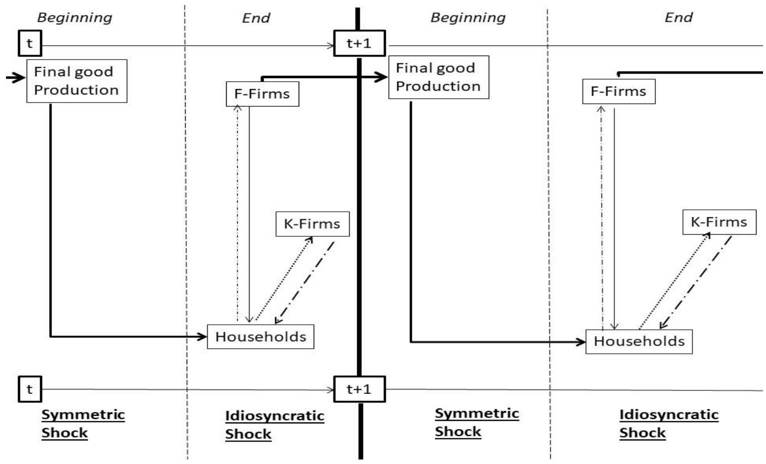

Figure 1: Flow of events in our economy.

In the model formulation we allow for growth and all variables are expressed in levels. For the existence of a BGP and for the detrended version of the model we remand to sections B and C in Appendix.

2.1 Final Good Producers

Monopolistically competitive retail firms assemble the final good bundle Y_t using a continuum of intermediate inputs Y_t^h. The representative firm profit maximization problem is:

$$\max_{Y_t, Y_t^h} P_t Y_t - \int_0^1 P_t^h Y_t^h dh$$

$$\text{s.t. } Y_t = \left[\int_0^1 \left(Y_t^h \right)^{\frac{\nu-1}{\nu}} dh \right]^{\frac{\nu}{\nu-1}}$$

From the first order conditions, we obtain:

$$Y_t^h = \left(\frac{P_t^h}{P_t} \right)^{-\nu} Y_t \tag{1}$$

$$P_t = \left[\int_0^1 \left(P_t^h \right)^{1-\nu} dh \right]^{\frac{1}{1-\nu}} \tag{2}$$

Price stickiness is based on the Calvo mechanism. In each period retail firm faces a probability $1 - \lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t^h = \pi_{t-1}^{\gamma_p} p_{t-1}^h \tag{3}$$

where $\gamma_p \in [0, 1]$ represents the degree of price indexation.

All the $1 - \lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$ the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t + s$ will read as $\widetilde{P}_t \left(\Pi_{t,t+s-1}^p \right)^{\gamma_p}$ where $\Pi_{t,t+s-1} = \pi_t ... \pi_{t+s-1} = \frac{P_{t+s-1}}{P_{t-1}}$. $\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \bar{\Lambda}_{t+s} \left(\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p} - P_{t+s} mc_{t+s} \right) Y_{t+s}^h$$

subject to:

$$Y_{t+s}^h = Y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\chi}}{P_{t+s}} \right)^{-\nu} \tag{4}$$

where Y_t^d is aggregate demand and $\bar{\Lambda}_t$ is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \bar{\Lambda}_{t+s} Y_{t+s}^d \left[\begin{array}{c} (1-\nu) \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{1-\nu} \widetilde{P}_t^{-\nu} \left(P_{t+s} \right)^{\nu} + \\ +\nu \widetilde{P}_t^{-\nu-1} P_{t+s}^{\nu+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{-\nu} \end{array} \right] = 0 \tag{5}$$

2.1.1 Intermediate good firms

Intermediate firms h are perfectly competitive and hire labor from households and exploit capital rented at the end of period $t-1$. Their production function is

$$Y_t^h = (z_t^n N_t)^{\chi} K_{t-1}^{1-\chi} \tag{6}$$

where N defines worked hours, K is the capital stock, z^n is a permanent labor augmenting technology shifter (LAT hereafter), such that $z_t^n = z_{t-1}^n g_{z,t}$ where

$$\ln(g_{z,t}) = (1-\rho_z)\ln(g_*) + \rho_z \ln(g_{z,t-1}) + \sigma^z \varepsilon_t^z \tag{7}$$

and $\varepsilon_t^z \sim N(0,1)$. As it can be seen, the labor augmenting technology shifter embeds a trending component, g_*, which is also the BGP gross rate of the economy.

Profits are as follows

$$\Pi_t^{fg} = mc_t Y_t^h - W_t N_t - r_{k,t} K_{t-1} \tag{8}$$

where W_t and $r_{k,t}$ respectively define the real wage and the rental rate of capital defined in consumption goods. Cost minimization implies

$$K_{t-1} = \frac{W_t}{r_{k,t}} \frac{(1-\chi)}{\chi} N_t$$

the real marginal costs are:

$$mc_t = \xi_t \left(\frac{r_{k,t}}{1-\chi}\right)^{1-\chi} \left(\frac{W_t}{z_t^n \chi}\right)^{\chi} \tag{9}$$

2.2 The Representative Household

We assume a standard characterization of households preferences,

$$U_t(C,N) = \sum_{i=0}^{\infty} \beta^{t+i} \left\{ \ln(C_{t+i} - aC_{t+i-1}) - \Phi \frac{N_{t+i}^{1+\theta}}{1+\theta} \right\} \tag{10}$$

Parameter a defines internal consumption habits. The flow budget constraint in real terms is

$$C_t + Q_t I_t + \frac{B_t}{P_t} = R_{b,t-1} \frac{B_{t-1}}{P_t} + r_{k,t} K_{t-1} + W_t N_t + \Pi_t^{K,F} \tag{11}$$

where Q is the relative price of investment goods, I defines investment goods, $\Pi^{K,F}$ are profits rebated by final and K-firms, and B is a nominally riskless bond of one-period maturity with gross remuneration R_b.[1] Households supply labor in the competitive labor market and choose consumption and savings, determining the accumulation of

[1] In this model there is no fiscal sector, therefore bonds are in zero net supply.

capital. When chosing the desired stock of capital to be used in next-period production, households transfer savings to K-producers who transform them into investment goods. We also assume that when assembling investment goods into the capital stock households bear investment adjustment costs. The law of motion of capital is

$$K_t = (1-\delta)K_{t-1} + \mu_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{12}$$

where δ is the capital depreciation rate, $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ defines investment adjustment costs, μ_t^i is a shock to the marginal efficiency of investment (i.e., MEI):

$$\ln\left(\mu_t^i\right) = \rho_{\mu^i} \ln\left(\mu_{t-1}^i\right) + \sigma^i \varepsilon_t^i.$$

and $\varepsilon_t^i \sim N(0,1)$ is an i.i.d. innovation term.

The FOCs are

$$\lambda_t = \left(C_t - aC_{t-1}\right)^{-1} - \beta a \left(C_{t+1} - aC_t\right)^{-1} \tag{13}$$

$$\frac{\Phi N_t^{\theta}}{\lambda_t} = W_t \tag{14}$$

$$Q_t = \varphi_t^k \mu_t^i \left\{1 - \left[S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}}\right)\right]\right\} + \\ +\beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\varphi_{t+1}^k \mu_{t+1}^i S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\} \tag{15}$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1}\left[\frac{r_{k,t+1}}{\varphi_t^k} + \frac{\varphi_{t+1}^k}{\varphi_t^k}(1-\delta)\right]\right\} \tag{16}$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1}\frac{R_{b,t}}{\pi_{t+1}}\right\} \tag{17}$$

Where $\varphi_t^k = \frac{\phi_t}{\lambda_t}$ is the shadow value of the capital stock in units of consumption goods, ϕ_t is the Lagrange multiplier of the law of motion of capital and $S'(\cdot) \equiv \gamma_I \left(\frac{I_{t+i}}{I_{t+i-1}} - 1\right)$.

2.3 The K-sector

At the end of period t, the generic K-sector firm j is characterized by the following production function

$$I_t^{K,j} = A_t^{K,j} \left(S_t^{K,j}\right)^{\alpha}$$

where $K = NE$, INC defines whether the firm is a new entrant or an incumbent one. $\alpha < 1$ implies that production occurs under decreasing returns to scale. $A_t^{K,j}$ is the idiosyncratic efficiency level. In terms of final goods, profits are

$$\Pi_t^{K,j} = Q_t I_t^{K,j} - S_t^{K,j} - f_t^K \tag{18}$$

where f_t^K is a fixed production cost such that $f_t^k = z_t^n f^k$, where we assume that K-firms fixed costs dynamics is governed by the LAT shifter, z_t^n. We also assume that, since K-producers idiosyincratic efficiency is fully observable by households, they are willing to transfer funds to K-firms if and only if they bear non-negative profits[2]. Therefore, the K-firm maximization problem boils down to a static one, and is solved maximizing profits with respect to $S_t^{K,j}$:

$$S_t^{K,j} = \left(Q_t \alpha A_t^{K,j}\right)^{\frac{1}{1-\alpha}} \tag{19}$$

Firms choose to operate if and only if their productivity level is such that they earn non-negative profits. The efficiency threshold can be worked out by plugging the optimal demand of savings within the zero-profits condition:

$$\hat{A}_t^K = \left(\frac{f_t^K}{1-\alpha}\right)^{1-\alpha} \frac{1}{Q_t \alpha^{\alpha}} \tag{20}$$

The K-firms cutoff is thus positively related to the fixed cost of production and is a negative function of the investment goods relative price. Thus entry and exit decisions are endogenous to any shock which affects Q_t.

At the end of time t our K-firms sector is made of a measure η_t of active firms distributed between new entrants, NE_t, and incumbents, INC_t, survived from period $t-1$.

$$\eta_t = NE_t + INC_t \tag{21}$$

[2]This is equivalent to assume that K-firms are not allowed to run into debt.

2.3.1 New Entrants

The market entry decision at the end of time t is conditional to firm NE, j idiosyncratic productivity level $A_t^{NE,j}$. A unit probability mass of potential NEs draw their individual $A_t^{NE,j}$ every period from a new and more efficient Pareto distribution[3]

$$f_t(A_t^{NE}) = \int_{e_t}^{+\infty} \frac{\gamma e_t^{\gamma}}{(A_t^{NE})^{\gamma+1}} d(A_t^{NE}) = 1 \quad with \quad A_t^{NE} \geq e_t \tag{22}$$

where γ is the tail index describing the distribution skewness, and $e_t = e_{t-1} g_{e,t}$ represents the technology frontier identifying the IST shock dynamics, where

$$\ln(g_{e,t}) = (1 - \rho_e)\ln(g_e) + \rho_e \ln(g_{e,t-1}) + \sigma^e \varepsilon_t^e \tag{23}$$

and $\varepsilon_t^e \sim N(0,1)$. Also the NEs technology frontier embeds a stochastic trend. Moreover, notice that, in order to ensure the existence of a BGP, $g_*^{1-\alpha} = g_e$ must hold true in a deterministic environment[4].

The mean of (22)

$$\mu\left(A_t^{NE}\right) = \frac{\gamma}{\gamma - 1} e_t. \tag{24}$$

is driven by the lower bound of the support defining the pdf which is increasing in the deterministic steady state.

The probability mass of effectively entering NE firms is obtained by cutting the pfd in (22) at the NEs threshold , $\hat{A}_t^{NE}$, obtained from eq. (20):

$$NE_t \equiv f_t(\hat{A}_t^{NE}) \equiv \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\gamma e_t^{\gamma}}{(A_t^{NE})^{\gamma+1}} d(A_t^{NE}) = \left[Q_t \alpha^{\alpha} e_t \left(\frac{1-\alpha}{f_t^{NE}}\right)^{1-\alpha}\right]^{\gamma} \tag{25}$$

2.3.2 Incumbents

At the end of period $t-1$ the mass of active K-firms, is

$$\eta_{t-1} = NE_{t-1} + INC_{t-1}$$

These K-firms produced the investment goods that allowed final goods firms to use the capital stock K_{t-1}. In period t only a fraction of the η_{t-1} K-firms

[3] The formulation of the potential NEs efficiency problem is a simplification of the one presented in Asturias et al. (2017)

[4] See section B.1 in Appendix.

choose to continue production, that is, those firms characterized by non negative profits. INC_{t-1} firms observe NE_{t-1} K-firms technological level and update their plants accordingly. For sake of simplicity, we posit that the updating process is stochastic, and we model it by assuming that INC_{t-1} firms draw their individual $A_t^{INC,j}$ from the Pareto distribution with support $\left[\hat{A}_{t-1}^{NE}, +\infty\right)$ that characterizes NE_{t-1} firms. Such a formulation brings the advantage of modeling endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent's efficiency.

Thus, at the beginning of period t, NEs and INCs, whose mass is η_{t-1}, are grouped into the Pareto pdf defining the idiosyncratic productivity level for each incumbent K-firm at the beginning of period t

$$f_t(A_t^{INC}) = \int_{\hat{A}_{t-1}^{NE}}^{+\infty} \frac{\gamma \left(\hat{A}_{t-1}^{NE}\right)^{\gamma}}{(A_t^{INC})^{\gamma+1}} \, d(A_t^{INC}) \tag{26}$$

but only firms that satisfy the non negative profits condition (20) will survive in the market. Thus the mass of INC_t is obtained as the fraction of η_{t-1} computed over the support share $\left[\hat{A}_t^{INC}, +\infty\right)$ of (26), where $\hat{A}_t^{INC} = \left(\frac{f^{INC_t}}{1-\alpha}\right)^{1-\alpha} \frac{1}{Q_t \alpha^{\alpha}}$ defines INC_t firms cutoff:

$$\begin{aligned} INC_t \equiv \eta_{t-1} f_t(\hat{A}_t^{INC}) \equiv \eta_{t-1} \int_{\hat{A}_t^{INC}}^{+\infty} \frac{\gamma \left(\hat{A}_{t-1}^{NE}\right)^{\gamma}}{(A_t^{INC})^{\gamma+1}} \, d(A_t^{INC}) = \\ = \eta_{t-1} \left[\hat{A}_{t-1}^{NE} \left(\frac{1-\alpha}{f_t^{INC}}\right)^{1-\alpha} Q_t \alpha^{\alpha}\right]^{\gamma} = \eta_{t-1} \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}}\right)^{1-\alpha} \frac{Q_t}{Q_{t-1}}\right]^{\gamma} \end{aligned}$$

which allows us to rewrite the mass of active firms as

$$\eta_t = \left[Q_t \alpha^{\alpha} e_t \left(\frac{1-\alpha}{f_t^{NE}}\right)^{1-\alpha}\right]^{\gamma} + \eta_{t-1} \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}}\right)^{1-\alpha} \frac{Q_t}{Q_{t-1}}\right]^{\gamma} \tag{27}$$

The mass of exit in t is thus

$$EXIT_t = \eta_{t-1} \left\{1 - \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}}\right)^{1-\alpha} \frac{Q_t}{Q_{t-1}}\right]^{\gamma}\right\} \tag{28}$$

2.3.3 K-firms production and the process of creative distruction

NEs and INCs supply functions are easily computed.

$$\begin{aligned} I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} \left(Q_t \alpha A_t^{NE}\right)^{\frac{\alpha}{1-\alpha}} dF(A_t^{NE}) \\ &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \\ &= \left[\alpha^\alpha e_t (1-\alpha)^{1-\alpha}\right]^\gamma \frac{\gamma}{\gamma(1-\alpha)-1} \frac{Q_t^{\gamma-1}}{(f_t^{NE})^{(1-\alpha)\gamma-1}} \end{aligned} \tag{29}$$

$$\begin{aligned} I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC} \left(Q_t \alpha A_t^{INC}\right)^{\frac{\alpha}{1-\alpha}} dF(A_t^{INC}) \\ &= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \\ &= \eta_{t-1} \left[\frac{\left(f_t^{NE}\right)^{1-\alpha}}{Q_{t-1}}\right]^\gamma \frac{\gamma}{\gamma(1-\alpha)-1} \frac{Q_t^{\gamma-1}}{(f_t^{INC})^{(1-\alpha)\gamma-1}} \end{aligned} \tag{30}$$

The details of the derivation are left in Appendix D.

Note that (29) and (30) show the source of "creative destruction" as only the former is directly affected by changes in e_t. For a deeper characterization of the "creative destruction" process we remand to Section 3.1.

2.4 Market clearing and policy rules

The K-goods and F-goods market clearing conditions respectively are

$$I_t = I_t^{NE} + I_t^{INC} \tag{31}$$

$$Y_t - NE_t f_t^{NE} - INC_t f_t^{INC} = C_t + S_t \tag{32}$$

where

$$S_t = \int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) + \int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \tag{33}$$

is the amount of input demanded for K-goods production coinciding with households savings and thus

$$I_t = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} \left(S\left(A_t^{NE}\right)\right)^{\alpha} dF(A_t^{NE}) + \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC} \left(S\left(A_t^{INC}\right)\right)^{\alpha} dF(A_t^{INC}) \quad (34)$$

At this point, for sake of completeness, we also work out accordingly the K-sector productivity, $\bar{A}_t$

$$\bar{A}_t = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} dF(A_t^{NE}) + \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC} dF(A_t^{INC}) \quad (35)$$

Finally, we assume that the Central Bank controls monetary policy by means of a simple Taylor rule with interest rate smoothing

$$\left(\frac{R_{b,t}}{R_b^{ss}}\right) = \left(\frac{R_{b,t-1}}{R_b^{ss}}\right)^{\rho_{R_b^{ss}}} \left[\left(\frac{\pi_t}{\pi^{ss}}\right)^{\kappa_\pi} \left(\frac{mc_t}{(\nu-1)/\nu}\right)^{\kappa_y} \exp\left\{\sigma^r \varepsilon_t^r\right\}\right]^{1-\rho_{R^{b^{ss}}}} \quad (36)$$

2.5 Calibration

We assume the BGP rate of the economy to be $g_* = 1.004$ on quarterly basis. For sake of simplicity we set the *NE*s fixed cost of entry at 1% of final output. Parameters calibration is meant to be on quarterly basis thus, in order to have $r_k^{ss} = 0.0351$, we impose $\beta = 0.99$. We calibrate Φ at a conventional value such that $N^{ss} = 0.3333$ and $\theta = 0.276$. Capital depreciation is also conventional and thus $\delta = 0.025$. We also set *NE*s as to be the 10% of total firms on annual basis in order to match the US business destruction rate as in Etro and Colciago (2010). Then, we normalize the steady state relative price of capital, $Q^{ss} = 1$, and the technology shifter in ss $z^n = 1$. For what concerns retailers, also these parameters have a standard calibration. The final goods elasticity of substitution, ν, is set equal to 11, the probability of not updating prices is $\lambda_p = 0.779$ and price indexation coefficient is $\gamma_p = 0.241$. The parameter governing investment adjustment costs is from estimates in JPT, $\gamma_I = 3.142$. Policy rule coefficients are also standard.

The last parameters to be calibrated are non conventional for the DSGE literature. They are the K-producers returns to scale, α, the *NE*s technology shifter, e^{ss}, and the tail index of the Pareto distribution, γ. Their calibration must be consistent with the fact that, from the law of motion of capital, $I_t^{ss} = 0.2049$ and consumption output ratio is $\approx 80\%$. In other words K-producers must be distributed in a way such that in equilibrium $Q^{ss} = 1$. To do this, we impose $\alpha = 0.8$

and $\gamma = 6.1$, this pins down the initial condition for the NEs technology shifter, $e^{ss} = 0.3397$. The calibration of α is at the lower bound of Basu and Fernald (1997) estimates, whilst the value assigned to γ is set to resemble Asturias et al. (2017)[5]. Finally, we set the interest rate smoothing coefficient, $\rho_{R_b^{ss}}$, equal to 0.8, and the MEI shock persistence, ρ_{μ^i}, equal to 0.813 as in JPT.

Parameters calibration is reassumed in Table 1.

Table 1: Parameters Calibration

Parameters	values	
g_*	1.004	Gross BGP rate
g_e	$g_*^{1-\alpha}$	Technology frontier BGP
β	0.994	Discount factor
a	0.815	Habit parameter
α	0.8	K-producers returns to scale
e^{ss}	0.3397	K-shifter initial condition
f^{NE}	0.01*Y	Entry Cost initial condition
$1 - H^{ss}$	0.025	Share of NEs over total K-firms
z^n	1	L-shifter initial condition
δ	0.025	Capital depreciation
γ	6.1	Tail index of K-firms distributions
χ	0.67	Labor share of income
N^{ss}	0.3333	SS labor
θ	0.276	Inverse Frisch elasticity of labor supply
ν	11	Final goods elasticity of substitution
λ_p	0.779	Probability of not updating prices
γ_p	0.241	Price indexation parameter
κ_π	1.5	Taylor Rule inflation coefficient
κ_y	0.125	Taylor Rule output coefficient
$\rho_{R_b^{ss}}$	0.8	Interest rate smoothing
γ_I	3.142	Investment adjustment costs
ρ_{μ^i}	0.813	MEI shock persistence
ρ_e	0	e_t growth persistence
ρ_z	0	z_t^n growth persistence
σ^e	0.05	e_t shock sd
σ^z	0.01	z_t^n shock sd
σ^i	0.05786	MEI shock sd

[5] In order to show that our results are not specific to particular parametrizations of the K-sector, we perform a sensitivity analysis of our key results in Section F.

3 Impulse Response Analysis

In this section we run some experiments by shocking the *NE*s efficiency draws, the labor augmenting technology shifter and the marginal efficiency of investment, respectively. The first two shocks have permanent effects, whilst the latter is persistent but stationary.

The NEs efficiency draws technology shock consists of a sudden and unexpected shift to the right of the potential *NE*s' pfd virtually keeping the *NE*s cutoffs (20) fixed. This causes an inflow of a higher mass of more productive *NE*s in the market strengthening competition among all K-firms. Indeed, it is theoretically consistent with what is known in the DSGE literature as the Investment Specific Technology shock (IST).

By contrast, the LAT shock triggers a completely different transmission mechanism by feeding the demand of capital which in turn lowers competition in the K-sector.

Finally, the MEI shock shares the same characteristics as documented in JPT augmented with a different K-sector dynamics typical of our formulation.

3.1 The IST shock

To begin with, we exploit steady state derivations in section B in Appendix to characterize the long run effects of the shock. Note that from conditions (81) and (82) it would be straightforward to show that the shock causes a permanent increase in the supply of investment goods which is associated to a permanent fall in their relative price. Numerical calculations show that a 5% white noise shock to the RW component of e_t causes a 6.91% increase in investment and a 4.63% fall in Q. This dynamics is broadly in line with the empirical evidence (see Greenwood et al., 1997 and Fisher, 2006). Technology diffusion through incumbents learning is crucial to determine this result. Given condition (73) this causes a fall in r_k and an increase in the capital labor ratio. As a result, in the medium run, higher wages determine an increase in the labor supply which then slowly comes back to its old steady state. Table 2 reports the steady state adjustments of key variables to a permanent 5% IST shock.

Let us now turn to the analyis of IRFs, i.e. Figures 2 and 3. The stochastic trend is added back in order to visualize the steady state transition from the old to the new steady state.

Given the stochastic process defining e_t, from condition (25) we know the IST shock implies an inflow of more productive *NE*s which shifts the supply schedule (29) to the right.

The fall in the relative price of investment goods raises the efficiency level of incumbent firms which is necessary to meet their zero-profit condition. As a result the mass of exiting incumbents increases. At the end of the initial period,

Table 2: Variables transition to 5% permanent shock to e_t

$Variable$	$\%\Delta\ from$ $initial\ ss$
Y	2.28
C	2.28
I	6.91
K	6.91
Q	-4.63
W	2.28
η	2.28

surviving incumbents adopt the new technology shifting to the right the supply of investment goods and further lowering the the relative price of investment goods. This fall begins to raise the NEs cutoff, gradually bringing down their number. New technology adoption gradually allows more incumbent firms to survive in the market, but this process is very slow. In fact, it takes 20 periods before INCs mass returns to the initial steady state level.

The total mass of active firms, η, shrinks for a few periods and then begins to pick up again. This "creative destruction" effect characterizes the early phase of the adjustment to the shock and is reinforced by the sluggish demand for investment goods. In fact, in spite of the immediate fall in Q, which calls for greater demand, the expectation of further reduction in their relative price induces households to postpone investment, which remains below the initial steady state value for 10 quarters. Consumption remains almost constant for the first 20 periods and then begins to pick up. Weak investment demand implies that capital goods producers initialy decrease their demand for final goods. As a result output immediately falls, initiating a moderate four-years-lasting recession. This pattern, in turn, drives the evolution of employment.

The results we have just shown are not specific of the DSGE version of the model as an RBC formulation yields virtually the same impulse responses.

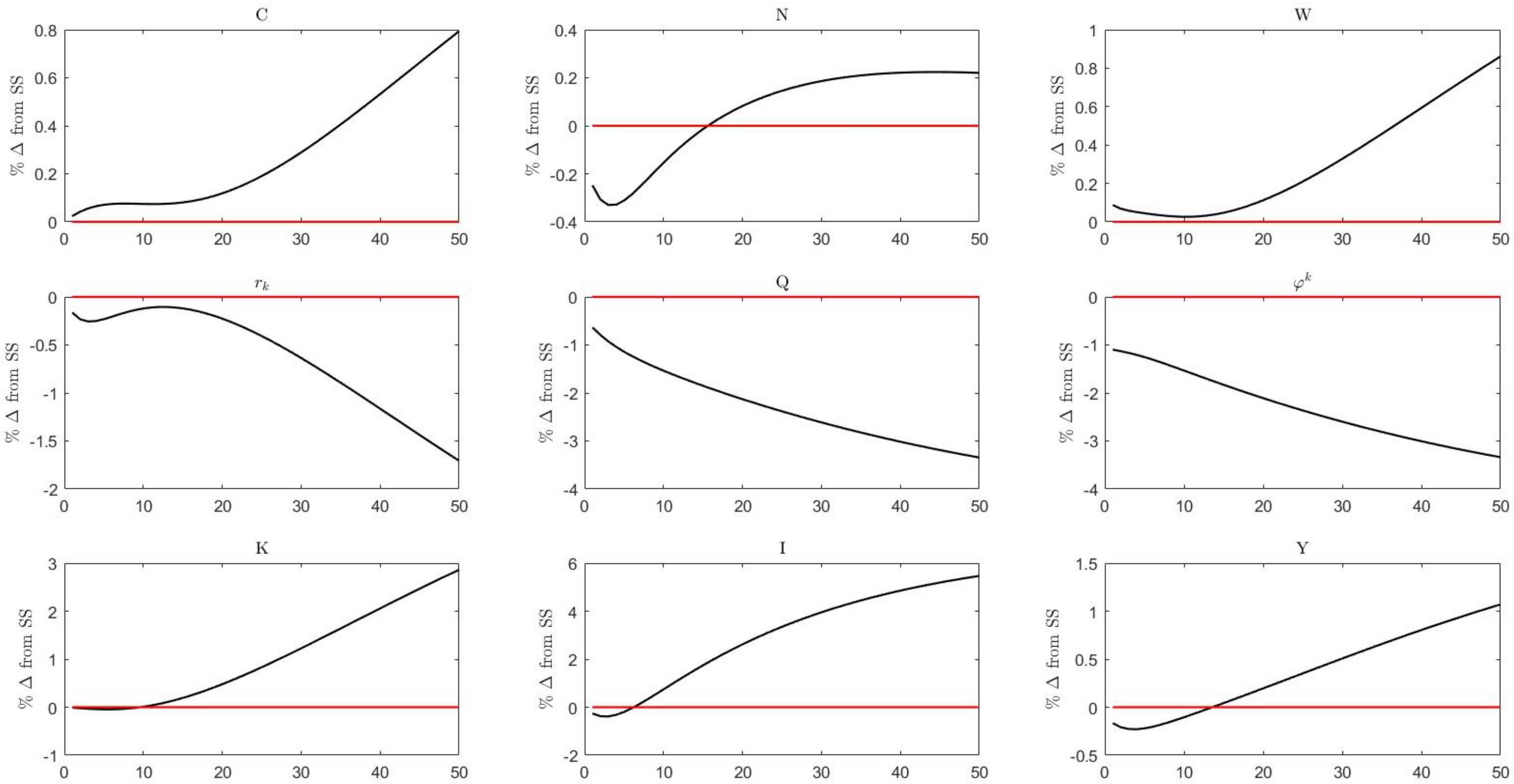

Figure 2: Impulse response functions to a permanent IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$, $\rho_e = 0$

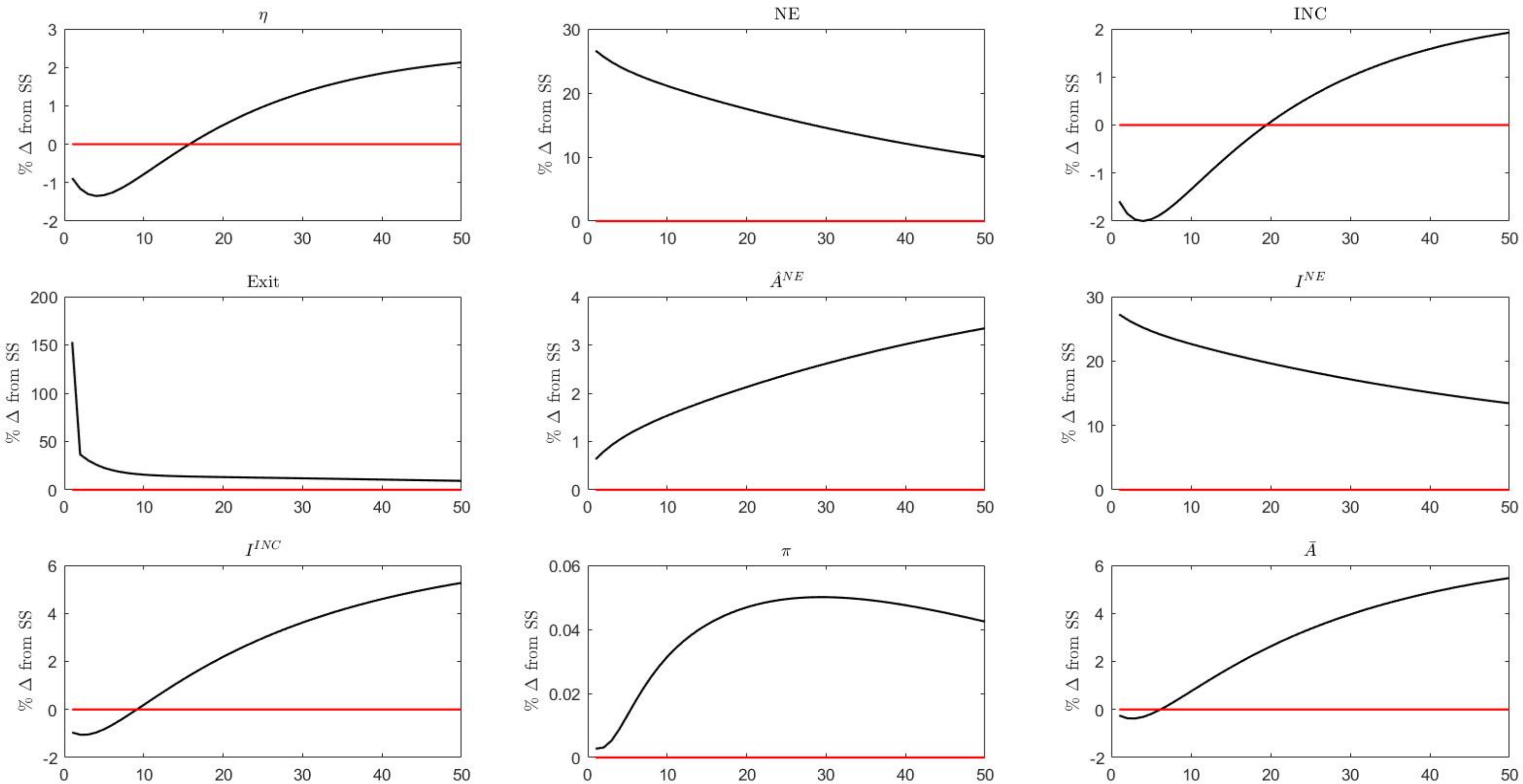

Figure 3: Impulse response functions to a permanent IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$, $\rho_e = 0$

3.1.1 The importance of knowledge spreading

As pointed out above, a pivotal role in this rich transmission mechanism is played by the incumbents updating ability which generates an impressive degree of persistence in real variable dynamics. To benchmark the importance of knowledge spreading, we have also constructed a version of this model where incumbents updating is not influenced by cyclical swings of the economy. To match the variance of final output obtained under our knowledge spillovers mechanism without any autoregressive component in $g_{e,t}$, we need to impose $\rho_e = 0.6137$ (i.e., the variance of $g_{e,t}$ is now 1.6 times larger than in the baseline model) when the incumbents updating process is acyclical.

For sake of clarity, stochastically detrended impulse responses relative to the above comparison are displayed in Figures 4 and 5.

The first thing to notice is that the updating mechanism introduces a very high degree of persistence in the model, indeed the recovery from creative destruction is far slower and the K-sector composition evolves pretty differently. However, it must be considered that the two stochastic trends have a quite distinct evolution as well because of the different values assigned to of ρ_e. Thus, should we add back the stochastic trend to impulse responses for the exogenous updating, the result would be qualitatively in line with Figures 2 and 3 but for the K-sector which would end up in a situation with permanently more NEs and less INCs.

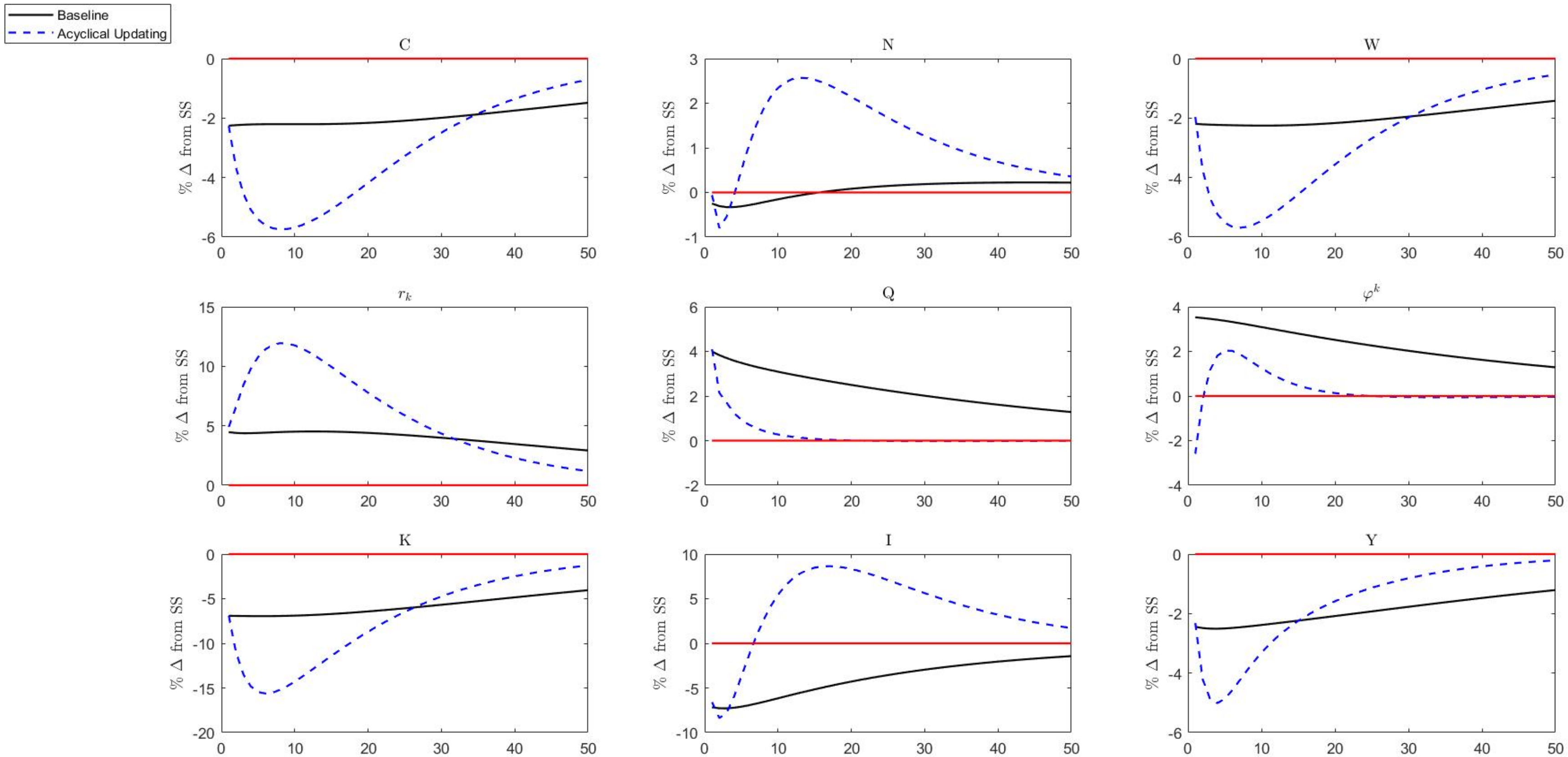

Figure 4: Impulse response functions to a stationary (stochastically detrended) IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$.
$\rho_e = 0$ under endogenous knowledge spreading
$\rho_e = 0.6137$ under exogenous knowledge spreading

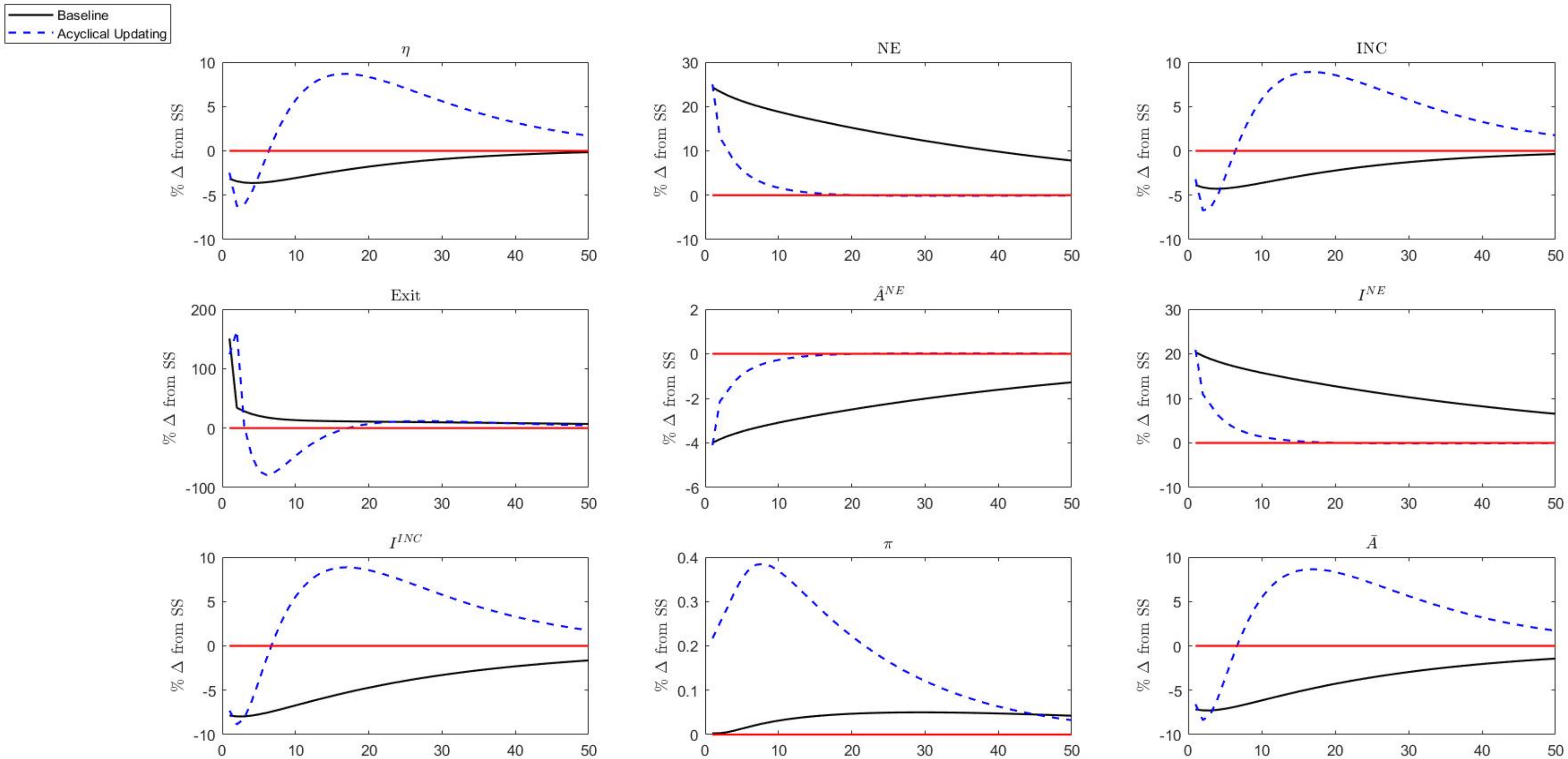

Figure 5: Impulse response functions to a stationary (stochastically detrended) IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$.
$\rho_e = 0$ under endogenous knowledge spreading
$\rho_e = 0.6137$ under exogenous knowledge spreading

3.2 A permanent increase in the labor augmenting technology shifter

The particular formulation of our model makes productivity in the K-sector react endogenously to any shock hitting the economy. In particular, the standard RBC-DSGE literature assumes that the dynamics of productivity is a fully exogenous process, here we try to overcome, at least partially, this view. Let us assume that for some reason there is a permanent white noise increase in the LAT shifter, z^n, and see how the transmission of this shock is affected by the endogenous updating at work in the K-sector. Impulse responses are shown in Figures 6 and 7, again the stochastic trend for each variable is added back.

As we can see, the permanent LAT shock still displays procyclical effects. First, let us consider the fact that this has a well known effect by increasing the demand of capital dictating a sudden expansion of the K-sector. However, it must be considered that there are some effects mitigating the above mechanism which are implicit in the embracement of nominal frictions in the DSGE model formulation. Indeed, its of common knowledge that sticky prices make labor demand shrink on impact as a response to a neutral technology improvement, thus weakening also the marginal product of capital and the associated demand of investment as compared to an RBC. From the other side the LAT shock also leads the dynamics of K-sector fixed costs which suddenly increases by killing both NEs and INCs. These mechanisms are both at work when z^n increases. The netting out of these overlapping effects explains the sudden drop in the mass of active firms. Then, on top of this, we observe an increase in the relative price of investment goods signalling an overall excess of demand, along with a sudden increase in final production. This not only allows for the gradual inflow of new less productive K-firms, but also makes less productive INCs survive. Therefore the K-industry ends up being made of more less efficient firms as both K-firms type thresholds are consistently lowered in the medium run. This mechanism downplays the quality of knowledge spillovers from NEs to INCs' as the impact of a true technological innovation in the K-sector is now absent. Notwithstanding the compositional effect of more inefficient survived firms is in favour of the K-sector total productivity increase. Therefore, in this case, the relative price of investments goods increases as well as K-sector productivity, $\bar{A}$, does, being thus in contrast with the IST shock case.

Finally we notice how the strange dynamics of K-sector cutoffs is due to the initial increase in the fixed costs as long as it is not overcome in magnitude by the increase in Q which lowers K-firms cutoffs and thus increases their mass.

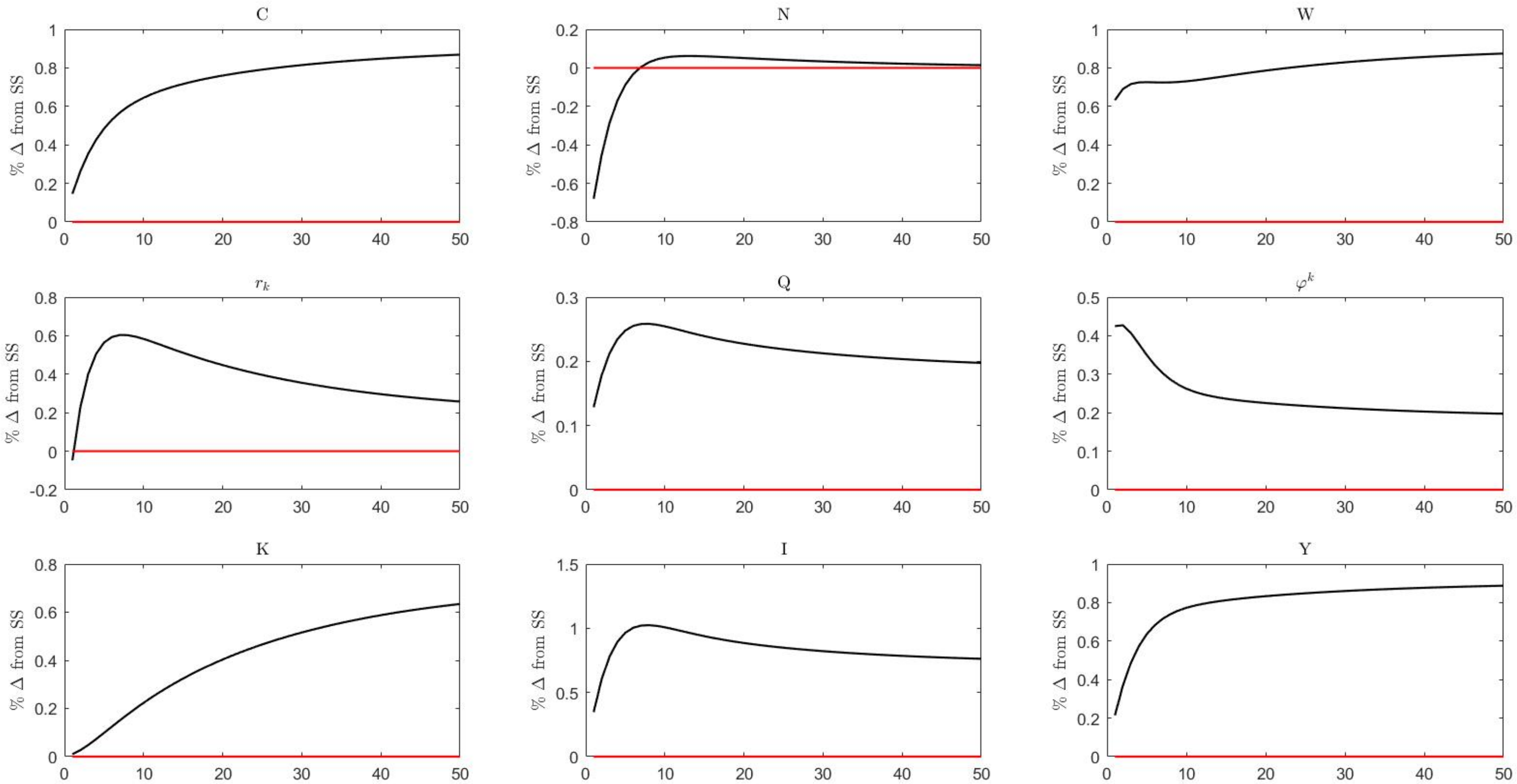

Figure 6: Impulse response functions to a Labor augmenting technology shifter permanent increase.
Shock size of the white noise component of $g_{z,t}$ is $\sigma^z = 0.01$.
$\rho_z = 0$

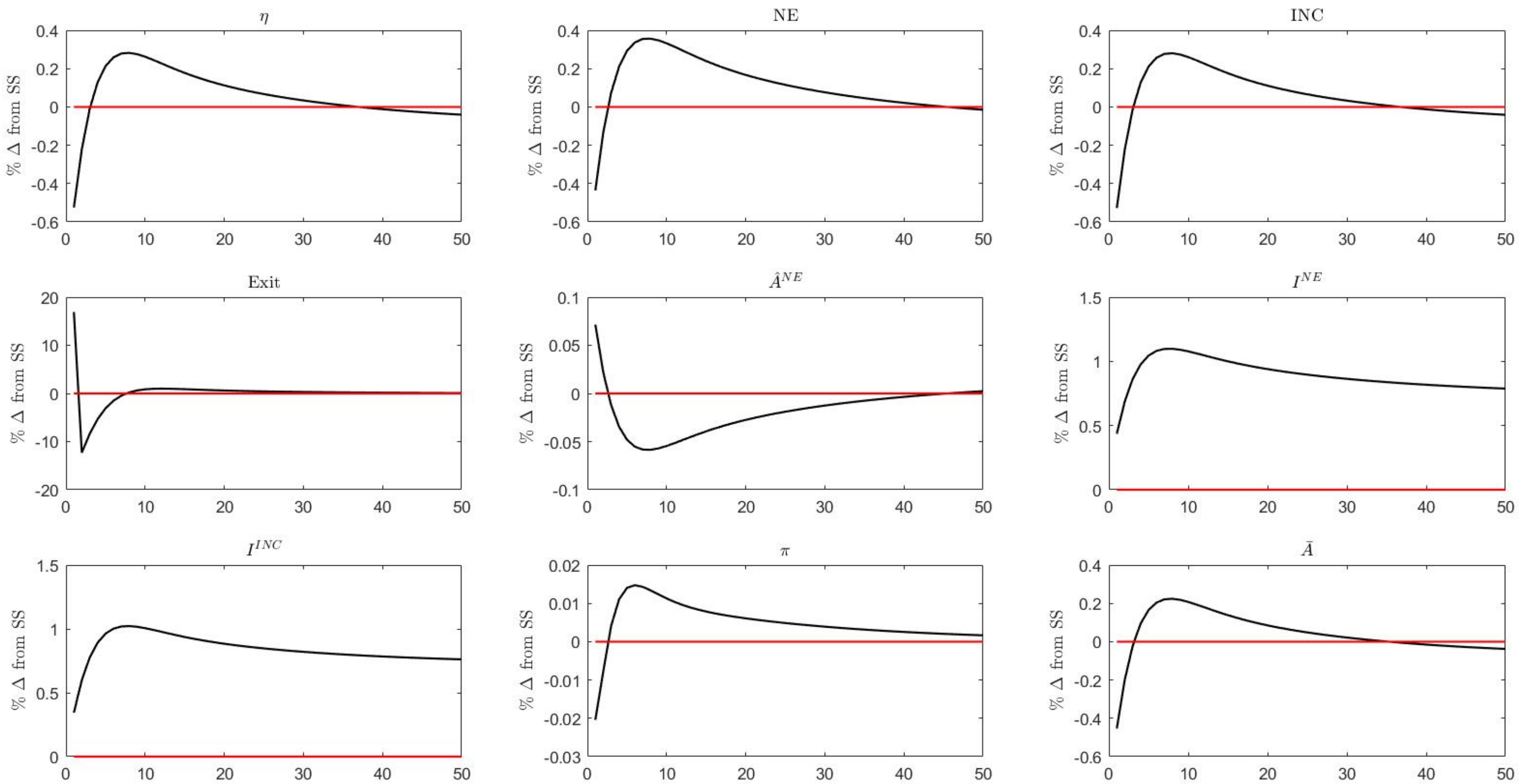

Figure 7: Impulse response functions to a Labor augmenting technology shifter permanent increase. Shock size of the white noise component of $g_{z,t}$ is $\sigma^z = 0.01$.
$\rho_z = 0$

3.3 MEI shock

We finally simulate a MEI shock of the kind documented in JPT by therefore assuming the same shock size. It must be noted however that we abstract from both capital utilization costs and sticky wages as compared to JPT. In this regard however, the introduction of our stylized K-sector sensibly enhances the shock transmission.

Impulse responses qualitatively resemble those in JPT, the transmission mechanism of the shock is not altered by our formulation of the K-industry. The MEI shock generates a persistent increase in the demand of capital and investment, this in turn lowers K-industry cutoffs. More inefficient K-firms survive and the production of investment goods increases as the composition effect prevails again in the K-industry. As in the permanent increase in the labor augmenting shifter, the relative price of investment goods and K-sector productivity comove positively. The persistent increase in capital accumulation in turn raises the demand of labor and thus final output. As usual consumption initially slightly decreases to leave room for the building up of investment to then start soaring when output peaks.

Further, we highlight how the relative price of investment, Q, and the shadow price of capital, φ^k, endogenously negatively comove as can be seen from (15). This is due to the fact that the MEI shock makes more convenient for households to buy investment goods as now their aggregation into capital is more efficient. This endogenous feedback from capital aggregation to the transformation of consumption into investment goods is completely neglected in JPT's analysis. Indeed, extending JPT to endogenous firms entry/exit and knowledge spillovers, triggers a strong procyclicality of Q in response to a positive MEI shock. This point is relevant as in the data the correlation between Q and Y is quite weak. This implies that the MEI shock cannot be accounted for the lion's share of business cycle variation as the generated volatility of Q is at odds with the empirical evidence.

Finally, must also be made clear that the MEI shock cannot replace the IST shock in accounting for counter-cyclical variations of the relative price of investment-goods as suggested in Greenwood et al. (1997) and extensively documented by Fisher (2006).

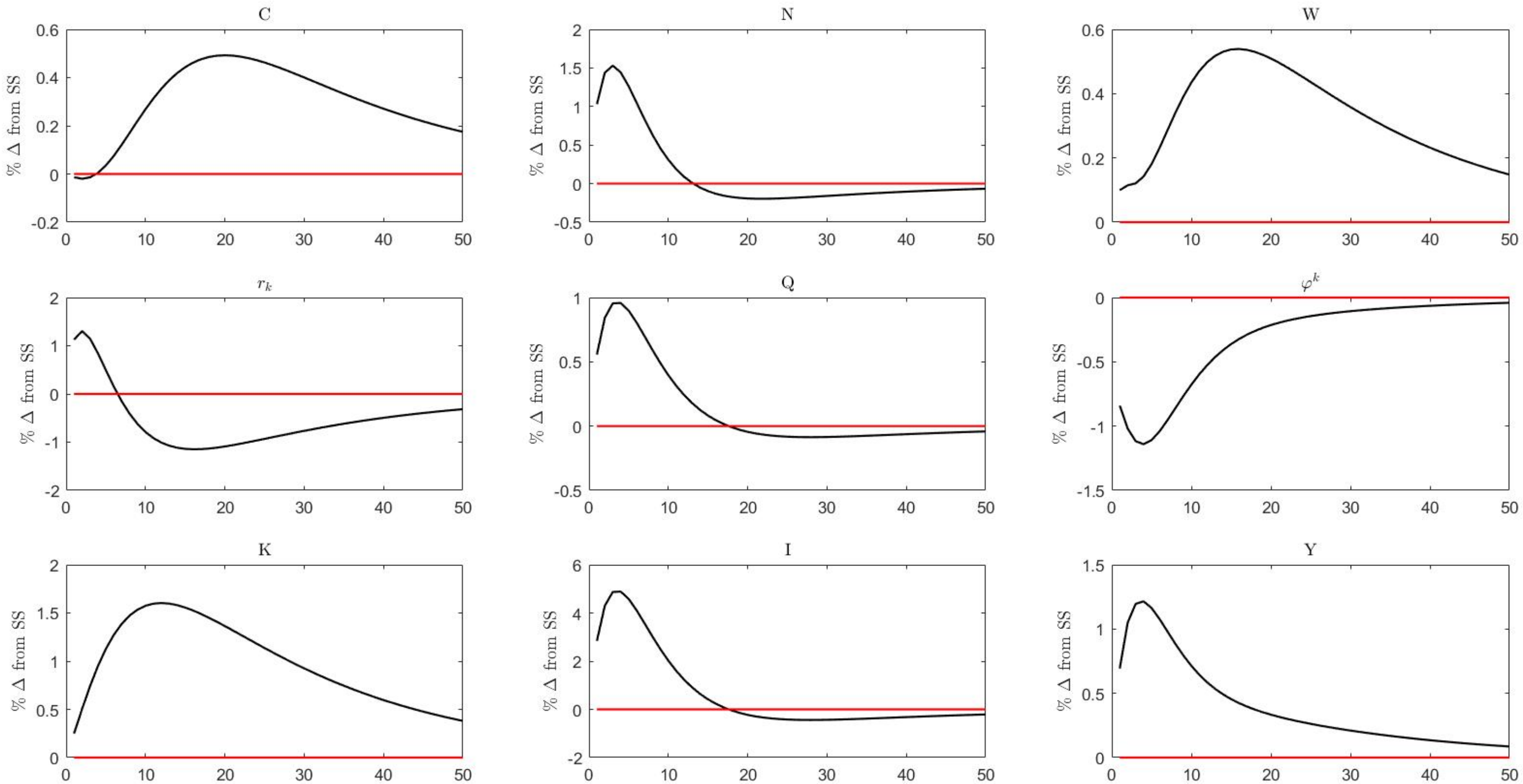

Figure 8: Impulse response functions to a MEI shock.
Shock size of the white noise component of μ_t^i is $\sigma^i = 0.05786$.
$\rho_{\mu^i} = 0.813$

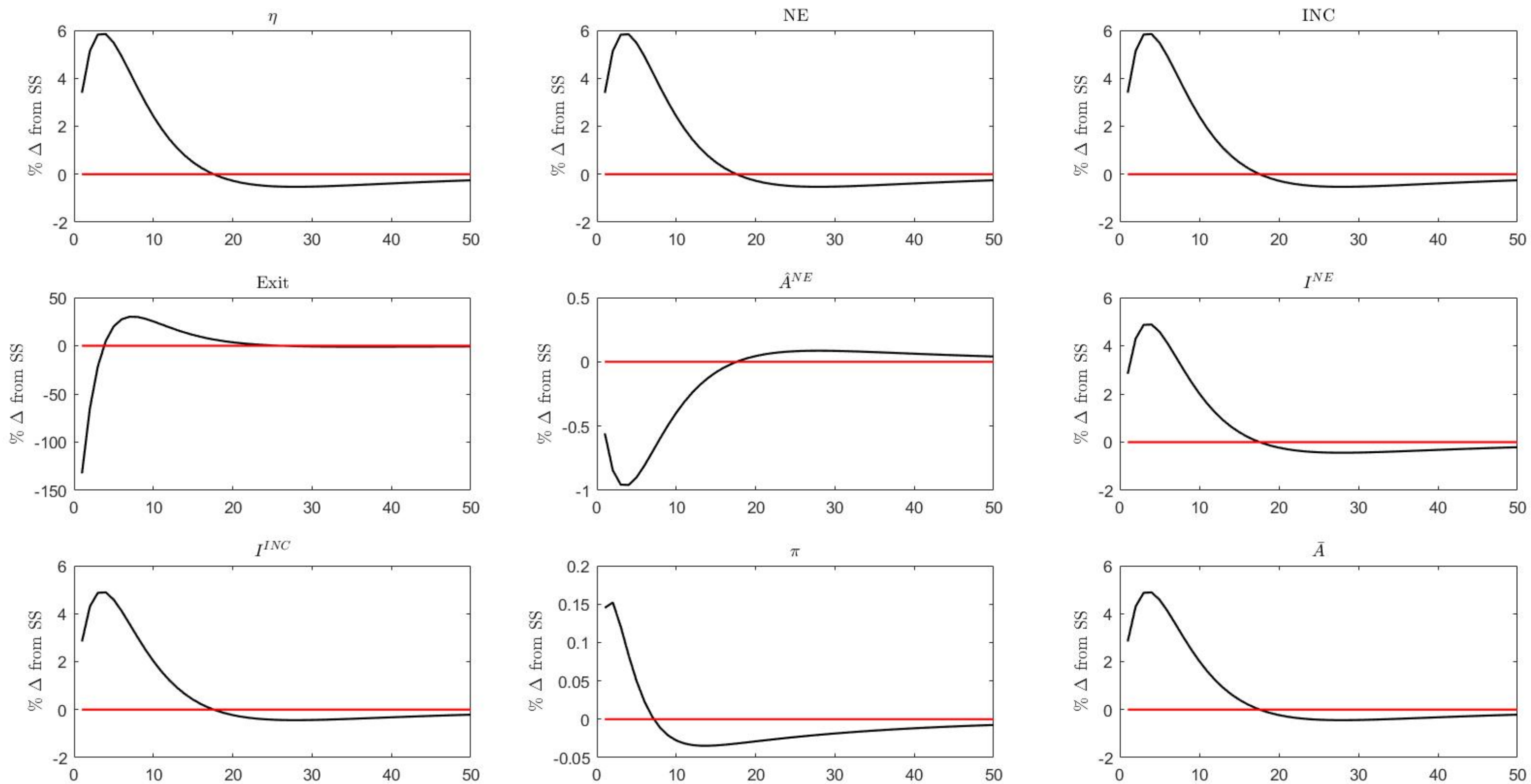

Figure 9: Impulse response functions to MEI shock.
Shock size of the white noise component of μ_t^i is $\sigma^i = 0.05786$.
$\rho_{\mu^i} = 0.813$

4 Conclusions

Our results are theoretically consistent with the permanent IST shock playing a major role in explaining both business cycle and long run movements of aggregated variables in the US.

From the theoretical perspective we constructed a novel two-sector model framed with a capital sector responding endogenously to any aggregate shock hitting the economy. In particular, we have that the relative price of investment-goods and sectoral productivity are two distinct objects and the sign of their comovement depends on the specific source of the shock under analysis. Indeed, it is the endogenous response of the relative price of investment, along with aggregate variables, which suggests that the IST permanent shock must play a prioritary role.

The IST shock propagation is magnified by the interaction of both endogenous firm entry-exit and knowledge spillovers. In this regard, when a technology progress materializes in the K-industry, it takes time to spread and display its full effects ("creative destruction") on aggregate variables, which however occur within business cycle frequencies.

From the other side, we have also documented how the same, endogenous, market forces giving raise to the above result seem to rule out a relevant role of the labor augmenting technology improvement as long run source of growth.

Last but not least, we have also seen as our model allows for endogenous feedbacks from the capital aggregation process to the transformation of consumption-goods into investment ones. This mechanism potentially downplays the relevance of the MEI shock as major business cycle driver since it implies a strong procyclicality of the relative price of investment goods which is not observed in the data.

References

Anzoategui, Diego, Diego Comin, Mark Gertler, and Joseba Martinez (2016). *Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence*. Tech. rep. National Bureau of Economic Research.

Asturias, Jose, Sewon Hur, Timothy J Kehoe, and Kim J Ruhl (2017). *Firm Entry and Exit and Aggregate Growth*. Tech. rep. National Bureau of Economic Research.

Basu, Susanto and John G Fernald (1997). "Returns to scale in US production: Estimates and implications". In: *Journal of political economy* 105.2, pp. 249–283.

Bilbiie, Florin O, Fabio Ghironi, and Marc J Melitz (2012). "Endogenous entry, product variety, and business cycles". In: *Journal of Political Economy* 120.2, pp. 304–345.

Canova, Fabio (2014). "Bridging DSGE models and the raw data". In: *Journal of Monetary Economics* 67, pp. 1–15.

Chatterjee, Satyajit and Russell Cooper (1993). *Entry and exit, product variety and the business cycle*. Tech. rep. National Bureau of Economic Research.

Clementi, Gian Luca and Berardino Palazzo (2016). "Entry, exit, firm dynamics, and aggregate fluctuations". In: *American Economic Journal: Macroeconomics* 8.3, pp. 1–41.

Colciago, Andrea and Lorenza Rossi (2015). "Firm Dynamics, Endogenous Markups, and the Labor Share of Income". In: *Macroeconomic Dynamics* 19.6, pp. 1309–1331.

Comin, Diego and Bart Hobijn (2010). "An Exploration of Technology Diffusion". In: *American Economic Review* 100.5, pp. 2031–59.

Comin, Diego A, Mark Gertler, and Ana Maria Santacreu (2009). *Technology innovation and diffusion as sources of output and asset price fluctuations*. Tech. rep. National Bureau of Economic Research.

Devereux, Michael B, Allen C Head, and Beverly J Lapham (1996). "Aggregate fluctuations with increasing returns to specialization and scale". In: *Journal of economic dynamics and control* 20.4, pp. 627–656.

Etro, Federico and Andrea Colciago (2010). "Endogenous market structures and the business cycle". In: *The Economic Journal* 120.549, pp. 1201–1233.

Fisher, Jonas DM (2006). "The dynamic effects of neutral and investment-specific technology shocks". In: *Journal of political Economy* 114.3, pp. 413–451.

Galí, Jordi (1999). "Technology, employment, and the business cycle: do technology shocks explain aggregate fluctuations?" In: *American economic review* 89.1, pp. 249–271.

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell (1997). "Long-run implications of investment-specific technological change". In: *The American Economic Review*, pp. 342–362.

— (2000). "The role of investment-specific technological change in the business cycle". In: *European Economic Review* 44.1, pp. 91–115.

Hopenhayn, Hugo A (1992). "Entry, exit, and firm dynamics in long run equilibrium". In: *Econometrica: Journal of the Econometric Society*, pp. 1127–1150.

Jaimovich, Nir and Max Floetotto (2008). "Firm dynamics, markup variations, and the business cycle". In: *Journal of Monetary Economics* 55.7, pp. 1238–1252.

Justiniano, Alejandro, Giorgio E Primiceri, and Andrea Tambalotti (2011). "Investment shocks and the relative price of investment". In: *Review of Economic Dynamics* 14.1, pp. 102–121.

Parente, Stephen L and Edward C Prescott (1994). "Barriers to technology adoption and development". In: *Journal of political Economy* 102.2, pp. 298–321.

Sims, Eric R (2011). "Permanent and transitory technology shocks and the behavior of hours: A challenge for DSGE models". In: *Unpublished manuscript. University of Notre Dame, Notre Dame, IN.*

A List of Detrended Equations

Here we list all the relevant equations in our model. The details of stochastic trend identification and remotion are left in section C.

The stochastic trend governing aggregate variables as Y_t, S_t, C_t, W_t and governing inflation dynamics recursive components is $\Gamma_t = e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} (z_t^n)^{1-\frac{\gamma(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}}$; the stochastic trend governing K_t and I_t is instead $\Lambda_t = e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} (z_t^n)^{1-\frac{\gamma(1-\alpha)}{1+\chi(\gamma-1)}}$.

The relative price of investment and the shadow price of capital in consumption units, Q_t and φ_t^k respectively, share the same stochastic trend that is $\frac{\Gamma_t}{\Lambda_t}$, which also determines the dynamics of $r_{k,t}$.

The stochastic trend governining K-firms mass is $\frac{e_t^{\gamma}(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma}}{\Lambda_t^{\gamma\chi}}$, whilst K-firms fixed costs trend is z_t^n.

Finally, the stochastic trend governing K-firms cutoffs is $\frac{\Lambda_t^{\chi}}{(z_t^n)^{\chi+\alpha-1}}$.

Lower case characters stand for stochastically detrended variables, the only exception concerns $r_{k,t}^*$, λ^*, and $\varphi_t^{*,k}$ which are stochastically detrended marginal utility of consumption and capital (in consumption units), and rental rate of capital.

A.1 Households

$$\lambda_t^* = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - a c_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - a c_t} \qquad \text{Marginal utility of consumption} \tag{37}$$

$$w_t = \frac{\Phi N_t^{\theta}}{\lambda_t^*} \qquad \text{Supply of Labor} \tag{38}$$

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\bar{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{\varphi_t^{*,k}} + \frac{\varphi_{t+1}^{*,k}}{\varphi_t^{*,k}}(1-\delta) \right] \right\} \qquad \text{Capital Euler} \tag{39}$$

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \frac{R_{b,t}}{\pi_{t+1}} \right\} \qquad \text{Bond Euler} \tag{40}$$

$$\begin{aligned} q_t = {} & \varphi_t^{*,k} \mu_t^i \left\{ 1 - \left[S'\left(\frac{i_t \bar{g}_t}{i_{t-1}} \right) \frac{i_t \bar{g}_t}{i_{t-1}} + S\left(\frac{i_t \bar{g}_t}{i_{t-1}} \right) \right] \right\} + \\ & + \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^* \bar{g}_{t+1}} \varphi_{t+1}^{*,k} \mu_{t+1}^i S'\left(\frac{i_{t+1}\bar{g}_{t+1}}{i_t} \right) \left(\frac{i_{t+1}\bar{g}_{t+1}}{i_t} \right)^2 \right\} \qquad \text{Investment rule} \end{aligned} \tag{41}$$

A.2 Intermediate Producers

$$k_t = (1-\delta)\frac{k_{t-1}}{\bar{g}_t} + \mu_t^i \left[1 - S\left(\frac{i_t \bar{g}_t}{i_{t-1}}\right)\right] i_t \qquad \text{Law of motion of capital} \qquad (42)$$

$$y_t = \frac{N_t^{\chi}\left(\frac{K_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi_t} \qquad \text{Final Output} \qquad (43)$$

$$r_{k,t}^* = \frac{mc_t}{\xi_t}(1-\chi)\left[\frac{\bar{g}_t N_t}{k_{t-1}}\right]^{\chi} \qquad \text{Demand of Capital} \qquad (44)$$

$$w_t = \frac{mc_t}{\xi_t}\chi\left[\frac{k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi} \qquad \text{Demand of Labor} \qquad (45)$$

A.3 Final Producers

$$d_t = \pi_t^* y_t + \beta\lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\pi_t^*}{\pi_{t+1}^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu-1} d_{t+1} \qquad \text{First Recursive Inflation Term} \qquad (46)$$

$$f_t = mc_t y_t + \beta\lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu} f_{t+1} \qquad \text{Second Recursive Inflation Term} \qquad (47)$$

$$d_t = \frac{\nu}{(\nu-1)} f_t \qquad \text{Inflation dynamics} \qquad (48)$$

$$1 = (1-\lambda_p)\left(\pi_t^*\right)^{1-\nu} + \lambda_p\left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{1-\nu} \qquad \text{Evolution of prices} \qquad (49)$$

$$\xi_t = (1-\lambda_p)\left(\pi_t^*\right)^{-\nu} + \lambda_p\left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{-\nu}\xi_{t-1} \qquad \text{Price Dispersion} \qquad (50)$$

A.4 Capital Producers

$$\hat{a}_t^{NE} = \left(\frac{f^{NE}}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_t \alpha^\alpha} \qquad \text{NEs cutoff} \qquad (51)$$

$$\hat{a}_t^{INC} = \left(\frac{f^{INC}}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_t \alpha^\alpha} \qquad \text{INCs cutoff} \qquad (52)$$

$$\eta_t^* = ne_t + inc_t \qquad \text{Mass of active K-producers} \qquad (53)$$

$$ne_t = \left(\frac{\hat{a}_t^{NE}}{e^{ss}}\right)^{-\gamma} \qquad \text{NEs mass} \qquad (54)$$

$$inc_t = \eta_{t-1}^* \left(\frac{\hat{a}_{t-1}^{NE} g_{z,t}^{\chi+\alpha-1}}{\hat{a}_t^{INC} \bar{g}_t^\chi}\right)^\gamma \qquad \text{INCs mass} \qquad (55)$$

$$exit_t = \left[1 - \left(\frac{\hat{a}_{t-1}^{NE} g_{z,t}^{\chi+\alpha-1}}{\hat{a}_t^{INC} \bar{g}_t^\chi}\right)^\gamma\right] \eta_{t-1}^* \qquad \text{Exit Mass} \qquad (56)$$

$$i_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (q_t\alpha)^{\frac{\alpha}{1-\alpha}} \qquad \text{NEs gross investment} \qquad (57)$$

$$i_t^{INC} = inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} (q_t\alpha)^{\frac{\alpha}{1-\alpha}} \qquad \text{INCs gross investment} \qquad (58)$$

$$i_t = i_t^{NE} + i_t^{INC} \qquad \text{K-producers gross investment} \qquad (59)$$

$$s_t = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha q_t \hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} + \\ + inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha q_t \hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} \qquad \text{K firms total input amount} \qquad (60)$$

$$\bar{a}_t = \frac{\gamma}{\gamma-1} ne_t \hat{a}_t^{NE} + \frac{\gamma}{\gamma-1} inc_t \hat{a}_t^{INC} \qquad \text{K-sector productivity} \qquad (61)$$

A.5 Market clearing conditions and policy rules

$$y_t = c_t + s_t + ne_t f^{NE} + inc_t f^{INC} \qquad \text{Market clearing} \tag{62}$$

$$\left(\frac{R_{b,t}}{R_b^{ss}}\right) = \left(\frac{R_{b,t-1}}{R_b^{ss}}\right)^{\rho_{R_b^{ss}}} \left[\left(\frac{\pi_t}{\pi^{ss}}\right)^{\kappa_\pi} \left(\frac{mc_t}{(\nu-1)/\nu}\right)^{\kappa_y} \exp\left\{\sigma^r \varepsilon_t^r\right\}\right]^{1-\rho_{R_b^{ss}}} \qquad \text{Taylor rule} \tag{63}$$

$$R_t = E_t\left\{\frac{R_{b,t}}{\pi_{t+1}}\right\} \qquad \text{Fisher equation} \tag{64}$$

A.6 Autoregressive processes

$$\ln(g_{e,t}) = (1-\rho_e)\ln(g_e) + \rho_e \ln(g_{e,t-1}) + \sigma^e \varepsilon_t^e \qquad \text{K-Tech growth rate} \tag{65}$$

$$\ln(g_{z,t}) = (1-\rho_z)\ln(g_*) + \rho_z \ln(g_{z,t-1}) + \sigma^z \varepsilon_t^z \qquad \text{L-Tech growth rate} \tag{66}$$

$$\ln(\bar{g}_t) = \frac{\gamma}{1+\chi(\gamma-1)}\ln(g_{e,t}) + \left[1 - \frac{\gamma(1-\alpha)}{1+\chi(\gamma-1)}\right]\ln(g_{z,t}) \qquad \text{K-Production stochastic growth} \tag{67}$$

$$\ln(\widetilde{g}_t) = \frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}\ln(g_{e,t}) + \left[1 - \frac{\gamma(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}\right]\ln(g_{z,t}) \qquad \text{F-Production stochastic growth} \tag{68}$$

$$\ln\left(\mu_t^i\right) = \left(1-\rho_{\mu^i}\right)\ln\left(\mu^i\right) + \rho_{\mu^i}\ln\left(\mu_{t-1}^i\right) + \sigma^i \varepsilon_t^i \qquad \text{MEI shock} \tag{69}$$

B Deterministic Steady State

In the deterministic steady state

$$\frac{z_t^n}{z_{t-1}^n} = g_* \tag{70}$$

where z^n defines the ss value of the technology shifter and g_* is the BGP growth rate of the economy. Output, capital, investment, consumption and the real wage all grow at the BGP rate while the relative price of investment and the labor supply are constant. The latter is pinned down by the preference parameter Φ in (14). Variables without time index are detrended or, in a deterministic environment, implicitly stationary and investment adjustment costs are nil. Further, from (18) it is straightforward to show that the fixed costs $f_t^{K,ss}$ also grows at the BGP rate g^*. We assume that the monetary policy rule achieves $\pi = 1$. As a result the real interest rate on the riskless bond is

$$\frac{g_*}{\beta} = R_b^{ss} \tag{71}$$

From condition (15) the shadow price of capital is equal to the price of investment goods

$$\varphi^{k,ss} = Q^{ss} \tag{72}$$

Q^{ss} is obtained when the investment goods market clears.

The steady state rental rate of capital is

$$\frac{g_*}{\beta} - 1 + \delta = \frac{r_k^{ss}}{Q^{ss}} \tag{73}$$

Then, for what concerns the final producer's capital FOC we have that in steady state

$$mc^{ss}(1-\chi)\left[\frac{g_* N^{ss}}{K^{ss}}\right]^{\chi} = r_k^{ss} \tag{74}$$

Demand of capital for production is

$$K^{ss} = g^* N^{ss}\left[\frac{mc^{ss}(1-\chi)}{\left(\frac{g_*}{\beta} - 1 + \delta\right) Q^{ss}}\right]^{\frac{1}{\chi}} \tag{75}$$

From the capital accumulation condition

$$I_t^{ss} = \left(1 - \frac{1-\delta}{g_*}\right) K^{ss}$$

Given the monopolistic nature of the final goods market, in the zero net inflation steady state the marginal cost is

$$mc^{ss} = \frac{\nu - 1}{\nu}$$

and the real wage is obtained solving

$$mc^{ss} = \left(\frac{r_k^{ss}}{1-\chi}\right)^{1-\chi} \left(\frac{W^{ss}}{g_* \chi}\right)^{\chi} \tag{76}$$

To obtain closed form solutions for the above conditions (72) - (76) we need to solve for the K-sector market clearing condition.

B.1 K-sector

To begin with, bear in mind that $f_t^{NE,ss}$, $f_t^{INC,ss}$, e_t^{ss} respectively define fixed costs and the K-firms technology shifter where the latter gows at the BGP rate $g_e \neq g_*$. We therefore define $e_t^{ss} \equiv e^{ss} g_e^t$ and $f_t^{NE,ss} \equiv f^{NE} g_*^t$ The solution for NE^{ss} is thus

$$NE^{ss} = \left[Q^{ss} \alpha^{\alpha} e^{ss} g_e^t \left(\frac{1-\alpha}{f^{NE} g_*^t}\right)^{1-\alpha} \right]^{\gamma} \tag{77}$$

Thus, in order to have a constant non zero and non diverging mass of NEs, it turns out that $g_e = g_*^{1-\alpha}$ must necessarily hold true.

Then we can rewrite (27) to obtain η^{ss}

$$\begin{aligned} \eta^{ss} &= NE^{ss} + INC^{SS} \\ \eta^{ss} &= NE^{ss} + \eta^{ss} \left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma} \\ \eta^{ss} &= \frac{NE^{ss}}{1 - \left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}} \end{aligned} \tag{78}$$

and INC^{ss}

$$INC^{ss} = NE^{ss} \frac{\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}{1 - \left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}$$

Further it must also be that $\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma} < 1$ in order to have a positive exiting mass of incumbents ruling thus out the possibility of an exploding mass of active firms as it can be seen from (28).

We can now solve for ss investments. From condition (29) we get:

$$I^{NE,ss} = NE^{ss}\frac{\gamma f^{NE,ss}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}} \tag{79}$$

and

$$I^{INC,ss} = INC^{ss}\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\hat{A}^{ss^{INC}}\right)^{\frac{1}{1-\alpha}}\left(Q^{ss}\alpha\right)^{\frac{\alpha}{1-\alpha}} \tag{80}$$

$$= \eta^{ss}\left[\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{1-\alpha}\right]^{\gamma}\frac{\gamma f^{INC}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}$$

As evident from above, when choosing K-firms returns to scale and tail index a condition must be respected, that is $\gamma(1-\alpha) > 1$. This is done to guarantee that gross investment production is positive as it appears clearly from (79) and (80).

B.2 Market clearing

Using (12), (70) and (75), we get

$$I^{ss} = \left(1-\frac{1-\delta}{g_*}\right)K^{ss}$$

$$= \left(1-\frac{1-\delta}{g_*}\right)g_* N^{ss}\left[\frac{\frac{\nu-1}{\nu}(1-\chi)}{\left(\frac{g_*}{\beta}-1+\delta\right)Q^{ss}}\right]^{\frac{1}{\chi}} \tag{81}$$

Using (77), (78), (79), (80), (81) we get that Q^{ss} solves the following market clearing condition for the investment goods sector:

$$I^{ss} = I^{NE,ss} + I^{INC,ss} \Rightarrow$$

$$\left(1-\frac{1-\delta}{g_*}\right)g_* N^{ss}\left[\frac{\frac{\nu-1}{\nu}(1-\chi)}{\left(\frac{g_*}{\beta}-1+\delta\right)Q^{ss}}\right]^{\frac{1}{\chi}} =$$

$$\left[\frac{e^{ss}Q^{ss}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\left(f^{NE}\right)^{1-\alpha}}\right]^{\gamma}\left\{\frac{\gamma f^{NE}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}} + \frac{\left[\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{1-\alpha}\right]^{\gamma}}{1-\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}\frac{\gamma f^{INC}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}\right\} \tag{82}$$

It is now possible to work out the closed form solutions for all endogenous variables.[6]

[6] As pointed out in section 2.5 we calibrate the model so that $Q^{ss} = 1$.

C Removing the Stochastic Trend Governing the Economy

Let us assume Γ_t is the stochastic trend governing Y_t, C_t, W_t and S_t, from which follows that, for instance, $Y_t = \Gamma_t y_t$, where smaller case characters are meant to be detrended variables if not differently specificated. Moreover, assume that Λ_t is the stochastic trend governing K_t and I_t, thus $K_t = \Lambda_t k_t$. We claim that both Γ_t and Λ_t are convolutions of the labor augmenting and the *NE*s permanent technology shifters, z_t^n and e_t.

C.1 Final production

Without any loss of generality we can rewrite final production as

$$y_t \Gamma_t = \frac{(z_t^n N_t)^{\chi} \left(\frac{\Lambda_t k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi} \tag{83}$$

where $\bar{g}_t = \frac{\Lambda_t}{\Lambda_{t-1}}$. Then we define

$$\Gamma_t = (z_t^n)^{\chi} \Lambda_t^{1-\chi} \tag{84}$$

Which can be interpreted as the non stationary stochastic evolution of TFP in our model. Then dividing (83) by (84) we obtain

$$y_t = \frac{(N_t)^{\chi} \left(\frac{k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi} \tag{85}$$

In a similar fashion we can work out the detrended law of motion of capital by dividing both sides by Λ_t that is

$$k_t = (1-\delta)\frac{k_{t-1}}{\bar{g}_t} + \mu_t^i \left[1 - \frac{\gamma_I}{2}\left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g}\right)^2\right] i_t \tag{86}$$

Without any loss of generality, the demand of capital can be rewritten as

$$r_{k,t} = \frac{mc_t}{\xi_t}(1-\chi)\left[\frac{z_t^n}{\Lambda_t}\frac{\bar{g}_t N_t}{k_{t-1}}\right]^{\chi} \tag{87}$$

Then, the stochastic trend leading $r_{k,t}$ is, exploiting (84), $\left(\frac{z_t^n}{\Lambda_t}\right)^{\chi} = \frac{\Gamma_t}{\Lambda_t}$, from which follows that the detrended rental rate of capital is

$$r_{k,t}^* = \frac{mc_t}{\xi_t}(1-\chi)\left[\frac{\bar{g}_t N_t}{k_{t-1}}\right]^{\chi} \tag{88}$$

where $r_{k,t}^* = r_{k,t}\frac{\Lambda_t}{\Gamma_t}$, is the stochastically detrended rental rate of capital.

Then we claimed that W_t shares the same stochastic trend as Y_t, therefore

$$w_t\Gamma_t = \frac{mc_t}{\xi_t}\chi\left(z_t^n\right)^{\chi}\Lambda_t^{1-\chi}\left[\frac{k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi}$$

implying that

$$w_t = \frac{mc_t}{\xi_t}\chi\left[\frac{k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi} \tag{89}$$

Where mc_t is stationary by itself since $\Gamma_t = (z_t^n)^{\chi}\Lambda_t^{1-\chi}$,

$$mc_t = \left(\frac{w_t\Gamma_t}{\chi z_t^n}\right)^{\chi}\left(\frac{r_{k,t}^*\Gamma_t}{(1-\chi)\Lambda_t}\right)^{1-\chi} \equiv \left(\frac{w_t}{\chi}\right)^{\chi}\left(\frac{r_{k,t}^*}{(1-\chi)}\right)^{1-\chi}$$

C.1.1 Retailers

For what concerns recursive inflation trend, they do have, by costruction, the same stochastic trend as Y. Therefore their detrended version is

$$d_t = \pi_t^* y_t + \beta E_t\left\{\lambda_p\frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\pi_t^*}{\pi_{t+1}^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu-1} d_{t+1}\right\} \tag{90}$$

In a similar fashion

$$f_t = mc_t y_t + \beta E_t\left\{\lambda_p\frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu} f_{t+1}\right\} \tag{91}$$

and thus

$$d_t = \frac{\nu}{\nu-1}f_t \tag{92}$$

Finally, price dispersion and price evolution are unchanged.

C.2 Households

From before we implicitly assumed $C_t = c_t\Gamma_t$, where we also define $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$. At this point we also have that $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$ where λ_t^* is the detrended MUC.

Then, MUC can be rewritten as

$$\frac{\lambda_t^*}{\Gamma_t} = \frac{1}{\Gamma_t c_t - a\Gamma_{t-1}c_{t-1}} - \beta a\frac{1}{\Gamma_{t+1}c_{t+1} - a\Gamma_t c_t}$$

Then multiplying on both sides by Γ_t and rearranging we have

$$\lambda_t^* = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - a c_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - a c_t} \tag{93}$$

Then the leisure-consumption relationship reads

$$w_t = \Phi \frac{N_t^\theta}{\lambda_t^*} \tag{94}$$

and from the Bond-Euler

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \frac{R_{b,t}}{\pi_{t+1}} \right\} \tag{95}$$

Before moving to the capital-Euler we remark that the shadow price of capital in consumption units is $\varphi_t^k = \frac{\phi_t^k}{\lambda_t}$ where we know that $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$ and must also hold true that $\phi_t^k = \frac{\phi_t^{*,k}}{\Lambda_t}$ since Λ_t is the stochastic trend governing capital. This implies that $\varphi_t^k = \frac{\phi_t^{*,k}/\Lambda_t}{\lambda_t^*/\Gamma_t}$. Plugging the latter into the capital-Euler yields

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{\varphi_t^{*,k}} \frac{\widetilde{g}_{t+1}}{\bar{g}_{t+1}} + \frac{\varphi_{t+1}^{*,k}}{\varphi_t^{*,k}} \frac{\widetilde{g}_{t+1}}{\bar{g}_{t+1}} (1-\delta) \right] \right\}$$

which boils down to

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\bar{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{\varphi_t^{*,k}} + \frac{\varphi_{t+1}^{*,k}}{\varphi_t^{*,k}} (1-\delta) \right] \right\} \tag{96}$$

The optimal investment condition implies that the stochastic trend of φ^k is the same leading Q implying $Q_t = q_t \frac{\Gamma_t}{\Lambda_t}$. Then dividing both sides by $\frac{\Gamma_t}{\Lambda_t}$ and rearranging we have

$$\begin{aligned} q_t = \varphi_t^{*,k} \mu_t^i & \left\{ 1 - \left[\gamma_I \left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g} \right) \frac{i_t \bar{g}_t}{i_{t-1}} + \frac{\gamma_I}{2} \left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g} \right)^2 \right] \right\} + \\ & + \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{\varphi_{t+1}^{*,k}}{\bar{g}_{t+1}} \mu_{t+1}^i \gamma_I \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} - \bar{g} \right) \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} \right)^2 \right\} \end{aligned} \tag{97}$$

C.3 K-firms

We remark that by assumption $f_t^k = z_t^n f^k$ for $k = [NE, INC]$. This allows us rewriting the NEs cutoff as

$$\hat{A}_t^{NE} = \left(\frac{z_t^n f^{NE}}{1-\alpha} \right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t \frac{\Gamma_t}{\Lambda_t}}$$

Then exploiting the fact that $\frac{\Lambda_t}{\Gamma_t} = \left(\frac{\Lambda_t}{z_t^n}\right)^{\chi}$ we have that

$$\hat{A}_t^{NE} = \hat{a}_t^{NE} \frac{\Lambda_t^{\chi}}{(z_t^n)^{\chi+\alpha-1}} \tag{98}$$

and thus

$$\hat{a}_t^{NE} = \left(\frac{f^{NE}}{1-\alpha}\right)^{1-\alpha} \frac{1}{\alpha^{\alpha} q_t} \tag{99}$$

from which follows

$$\hat{a}_t^{INC} = \left(\frac{f^{INC}}{1-\alpha}\right)^{1-\alpha} \frac{1}{\alpha^{\alpha} q_t} \tag{100}$$

At this point we can easily rewrite the mass of active NEs as

$$NE_t = \left(\frac{e^{ss} e_t \left(z_t^n\right)^{\chi+\alpha-1}}{\hat{a}_t^{NE} \Lambda_t^{\chi}}\right)^{\gamma} = \left(\frac{e^{ss}}{\hat{a}_t^{NE}}\right)^{\gamma} \frac{(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma}}$$

implying that $NE_t = ne_t \frac{(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma}}$ and so

$$ne_t = \left(\frac{e^{ss}}{\hat{a}_t^{NE}}\right)^{\gamma} \tag{101}$$

By BGP conditions we know that also η_t and INC_t share the same stochastic trend as NE_t, this implies

$$\eta_t^* = ne_t + inc_t \tag{102}$$

and

$$inc_t = \eta_{t-1}^* \left(\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \frac{(g_t^*)^{\chi+\alpha-1}}{\bar{g}_t^{\chi}}\right)^{\gamma} \tag{103}$$

where of course $\eta_t^* = \eta_t \frac{(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma}}$.

C.4 Stochastic Trends Identification

Notice that from the aggregate resource constraint in (32) it turns out that the stochastic trend leading $NE_t f_t^{NE}$ and $INC_t f_t^{INC}$ mus be, by construction, the same leading Y_t, C_t and S_t, i.e. Γ_t. Then, given that $NE_t z_t^n f^{NE} \equiv ne_t f^{NE} \frac{(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma}} z_t^n$, we can easily work out

$$\Gamma_t = \frac{(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma+1} \, e_t^{\gamma}}{\Lambda_t^{\chi\gamma}} \tag{104}$$

Then, plugging the relationship $\Gamma_t = (z_t^n)^{\chi} \, \Lambda_t^{1-\chi}$ into (104) allows identifying the stochastic trend leading both K_t and I_t, that is

$$\Lambda_t = e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} \, (z_t^n)^{1-\frac{\gamma(1-\alpha)}{1+\chi(\gamma-1)}} \tag{105}$$

Then, plugging (105) into (104) we have

$$\Gamma_t = e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} \, (z_t^n)^{1-\frac{\gamma(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}} \tag{106}$$

Which is the stochastic trend leading aggregate variables but K_t and I_t. Thus the stochastic trend governing aggregate variables is a Cobb-Douglas of the the permanent shifter governing the K-sector and final goods production [7].

At this point we can also identify the stochastic trend leading K-firms cut-off. For instance substituing for (105) into (98) we obtain that the corresponding stochastic trend is $e_t^{\frac{\gamma\chi}{1+\chi(\gamma-1)}} \, (z_t^n)^{\frac{(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}}$.

Finally, plugging (105) into (101), (102) and (103) it turns out that the stochastic trend leading the K-firms industry composition is $e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} \, (z^n)^{-\frac{\gamma(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}}$.

C.5 K-firms production

At this point, since we claimed that K_t and I_t are governed by the same stochastic trend, i.e. Λ_t, this implies that $I_t^{NE} = i_t^{NE}\Lambda_t$. Then

$$i_t^{NE}\Lambda_t = ne_t \frac{e_t^{\gamma} \, (z_t^n)^{\gamma\chi+\gamma\alpha-\gamma}}{\Lambda_t^{\chi\gamma}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\frac{\hat{a}_t^{NE} \Lambda_t^{\chi}}{(z_t^n)^{\chi+\alpha-1}} \right)^{\frac{1}{1-\alpha}} \left[\alpha q_t \left(\frac{z_t^n}{\Lambda_t} \right)^{\chi} \right]^{\frac{\alpha}{1-\alpha}}$$

From which rearranging

$$i_t^{NE}\Lambda_t = ne_t \frac{e_t^{\gamma} \, (z_t^n)^{\gamma\chi+\gamma\alpha-\gamma}}{\Lambda_t^{\chi\gamma}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (\alpha q_t)^{\frac{\alpha}{1-\alpha}} \, \Lambda_t^{\chi} \, (z_t^n)^{1-\chi}$$

but from (105) we know that $\Lambda_t^{1+\chi(\gamma-1)} = e_t^{\gamma} \, (z_t^n)^{1+\chi(\gamma-1)-\gamma(1-\alpha)}$

Which therefore implies

$$i_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (\alpha q_t)^{\frac{\alpha}{1-\alpha}} \tag{107}$$

[7] According to our parametrization $\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)} < 1$.

and thus it must also be that

$$i_t^{INC} = inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} (\alpha q_t)^{\frac{\alpha}{1-\alpha}} \tag{108}$$

and

$$i_t = i_t^{NE} + i_t^{INC} \tag{109}$$

C.6 Aggregate Resources Constraint

There is only one variable to be detrended yet. By construction it must be $S_t = s_t\Gamma_t$ which also implies $s_t\Gamma_t = s_t^{NE}\Gamma_t + s_t^{INC}\Gamma_t$.

Then it is sufficient to show that

$$s_t^{NE}\Gamma_t = ne_t \frac{e_t^{\gamma}\left(z_t^n\right)^{\gamma\chi+\gamma\alpha-\gamma}}{\Lambda_t^{\chi\gamma}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \left(\frac{z_t^n}{\Lambda_t}\right)^{\chi} \hat{a}_t^{NE} \left(\frac{\Lambda_t}{z_t^n}\right)^{\chi} \left(z_t^n\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$$

can be rewritten as

$$s_t^{NE}\Gamma_t = ne_t \frac{e_t^{\gamma}\left(z_t^n\right)^{\gamma\chi+\gamma\alpha-\gamma+1}}{\Lambda_t^{\chi\gamma}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \hat{a}_t^{NE}\right]^{\frac{1}{1-\alpha}}$$

and again, since $\Lambda_t^{1+\chi(\gamma-1)} = e_t^{\gamma}\left(z_t^n\right)^{1+\chi(\gamma-1)-\gamma(1-\alpha)}$, it must be that

$$s_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \hat{a}_t^{NE}\right]^{\frac{1}{1-\alpha}} \tag{110}$$

and

$$s_t^{INC} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \hat{a}_t^{INC}\right]^{\frac{1}{1-\alpha}} \tag{111}$$

and

$$s_t = s_t^{NE} + s_t^{INC} \tag{112}$$

Thus we have shown that

$$y_t - ne_t f^{NE} - inc_t f_t^{INC} = c_t + s_t \tag{113}$$

holds true.

C.7 Stochastic Growth Rates Identification

We claimed that $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$, then exploiting (106) it turns out that

$$\widetilde{g}_t = g_{e,t}^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} g_{z,t}^{1-\frac{\gamma(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}} \tag{114}$$

meaning that the stochastic BGP growth rate is a convolution of the stochastic growth rate of e_t and z_t^n. Similarly for $\frac{\Lambda_t}{\Lambda_{t-1}} = \bar{g}_t$ it follows that

$$\bar{g}_t = g_{e,t}^{\frac{\gamma}{1+\chi(\gamma-1)}} g_{z,t}^{1-\frac{\gamma(1-\alpha)}{1+\chi(\gamma-1)}} \tag{115}$$

Finally in the deterministic steady state we have that $g_e = g_*^{1-\alpha}$ and that $\bar{g} = \widetilde{g} = g_*$ must hold true, i.e. the deterministic BGP is the same for all aggregated variables, which is verified plugging $g_e = g_*^{1-\alpha}$ into the deterministic formulation of (114) and (115).

D K-sector Production

Here we derive overall production in the K-sector.

D.1 Derivation of NEs total production

Let us start from new entrants. We know that the production function for the generic NE firm can be expressed as

$$I_t^{NE,j} = \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t \alpha)^{\frac{\alpha}{1-\alpha}} \tag{116}$$

Then, by exploiting the transformation theorem we can compute the expected value of NEs production

$$\begin{aligned} I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \, dF(A_t^{NE}) \\ \Rightarrow I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} f(A_t^{NE}) \, d(A_t^{NE}) \\ \Rightarrow I_t^{NE} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \gamma (e_t/\varphi)^{\gamma} \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE}\right)^{\frac{1}{1-\alpha}-\gamma-1} \, d(A_t^{NE}) \\ \Rightarrow I_t^{NE} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \gamma (e_t/\varphi)^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_t^{NE}\right)^{\frac{1}{1-\alpha}-\gamma} \right]_{\hat{A}_t^{NE}}^{+\infty} \\ \Rightarrow I_t^{NE} &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{117}$$

Where we exploited the fact that $NE_t = \left(\frac{\hat{A}_t^{NE}}{e_t}\right)^{-\gamma}$ and by assumption it must hold true that $\gamma(1-\alpha) - 1 > 0$.

Notice moreover that what we have computed is nothing different than the mean of a truncated distribution without normalizing it to a unit probability measure i.e., without dividing it by the probability share over which it is computed. This is done because in our model we want a measure of the total production in the NEs industry. Should one want to compute the idiosyncratic average production it would be sufficient dividing (117) by NE_t.

D.2 Derivation of INCs total production

Let us repeat the same computation for incumbents. The production function for the generic incumbent firm is

$$I_t^{INC,j} = \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \tag{118}$$

Then, as before we have

$$\begin{aligned}
I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \, dF(A_t^{INC}) \\
\Rightarrow I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \, f(A_t^{INC}) \, d(A_t^{INC}) \\
\Rightarrow I_t^{INC} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \gamma\eta_{t-1} \left(\hat{A}_{t-1}^{NE}\right)^{\gamma} \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC}\right)^{\frac{1}{1-\alpha}-\gamma-1} \, d(A_t^{INC}) \\
\Rightarrow I_t^{INC} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \gamma\eta_{t-1} \left(\hat{A}_{t-1}^{NE}\right)^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_t^{INC}\right)^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t^{INC}}^{+\infty} \\
\Rightarrow I_t^{INC} &= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{119}$$

Where we have exploited the fact that $\eta_{t-1}\left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{INC}}\right)^{\gamma} = INC_t$.

Then, as the expected value of the sum is the sum of the expected values, we have that

$$I_t = I_t^{NE} + I_t^{INC} \tag{120}$$

E K-firms profits derivation

Total revenues of the K-sector are thus $Q_t I_t$. Let us now define the the total amount of savings employed as input in the production of capital goods as

$$S_t = \int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) + \int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \tag{121}$$

where $\int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) \equiv S_t^{NE}$ and $\int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \equiv S_t^{INC}$ are the total amount of inputs used in NEs and INCs sector production.

It follows that profits are respectively

$$\begin{aligned} \Pi_t^{NE} &= Q_t I_t^{NE} - S_t^{NE} - NE_t f^{NE} \\ &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(Q_t \hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - NE_t f^{NE} \end{aligned} \tag{122}$$

and

$$\begin{aligned} \Pi_t^{INC} &= Q_t I_t^{INC} - S_t^{INC} - INC_t f^{INC} \\ &= INC_t \frac{\phi(1-\alpha)}{\phi(1-\alpha)-1} \left(Q_t \hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - INC_t f^{INC} \end{aligned} \tag{123}$$

Which are always positive by construction as $\alpha < 1$. Then, define the total expenditures in fixed costs of the K-sector as

$$\overline{F}_t = NE_t f^{NE} + INC_t f^{INC} \tag{124}$$

Finally let us define the total amount of profits in the K-sector as

$$\overline{\Pi}_t = \Pi_t^{NE} + \Pi_t^{INC} \tag{125}$$

Then by substituting for (31), (122), (123) and (124) into equation (125) and rearranging we obtain the following identity

$$\overline{\Pi}_t + S_t + \overline{F}_t = Q_t I_t \tag{126}$$

Simply stating that the total amount of capital goods (in real terms) produced in the K-sector must be equal to the sum of profits, the input share of production and the total amount of fixed costs. Indeed households hold the stock of capital and use savings to buy the gross investment from the K-sector as a whole. The share of households revenues S_t is employed by K-firms in investment production at real good price cost, whilst the share $\overline{F}_t$ is devoted to fixed costs payment.

F Sensitivity analysis for different parametrizations of the K-sector

Here we perform a sensitivity analysis for possibly different parametrizations of of our K-sector. The key parameters we play around with are the pareto tail index, γ, and the *NE*s initial condition for the technological shifter, e. The first thing to notice is that varying K-firms returns to scale, α is useless. This is can be seen by plugging (141) and (142) into (147) and (148), respectively. It turns out that the dynamics of gross investments, and thus of other real variables, is never affected by changes in α.

Thus, the crucial parameter for our sensitivity analysis is γ. In particular it describes the rate at which the updated incumbents pdf decays. The lower it is, the slower the pdf approaches zero. This also implies that incumbents updating is more successful since, as compared with higher values of γ, there are more frequencies for higher idiosyncratic productivity values. It follows that the recovery from creative destruction is faster for lower values of γ. We run our model for three different values of γ, in order to let the market clear at any different value of γ is associated a different ss value for the *NE*s technology shifter, e, is needed. The table below shows how the lower bound of the potential *NE*s support, e, changes for different values of γ considered in the sensitivity analysis.

Table 3: Different Tail Indexes Calibration

γ	e^{ss}
6	0.3401
9	0.4713
12	0.5228

Figures 10 and 11 show how impulses to a permanent IST shock with respect to different values of γ, the shocks magnitude is the same as before. For what concerns macro variables dynamics, they are slightly affected. Where the sensititvity is crucial is for K-sector specific variables. In particular, we can see how the lower is γ, the less *NE*s enter the market. This is because a more diffuse potential *NE*s distribution implies that there are more potential *NE*s on frequencies lower than $\hat{A}_t^{NE}$ who therefore remain out of the market. A specular reasoning applies to a lower mass of exit among updated incumbents.

Figures 12 and 13 show instead impulses to a permanent LAT shock. Also in this case the most affected variables are those specific to the K-sector, even if now gross investment, and thus capital, and Q_t dynamics are more reactive to changes in γ. This is why the lower is γ, the less *NE*s enter and the less *INC*s, who would have otherwise died, remain in the business. On aggregate the K-sector ends up being smaller for lower values of γ, and thus the relative price of investment reacts by increasing more strongly to clear the market. In the end, this calls for a smaller increase in the demand of investment goods.

Finally, Figures 14 and 15 show the same sensitivity analysis for the transitory MEI shock. In this case higher values of γ are associated with higher variations in final output. This is may seem counterintuitive as in principle all K-firms are on average less productive, hence are more concentrated towards the fat tail of the distribution. In addition the elasitcity of Q (and thus the one of $\hat{A}^k$) is lower for higher values of γ. However, the gain in the probability mass defining the K-sector dimension is higher when the pareto is more right-skewed and contingently the K-sector thresholds lower. This implies a more responsive production of investment goods eventhough the increase in Q is more muted. Then, capital accumulation, and thus the increase in the demand of labor, is fastened.

In general, however, our results are virtually unaffected with respect to different reasonable parametrizations of the K-industry.

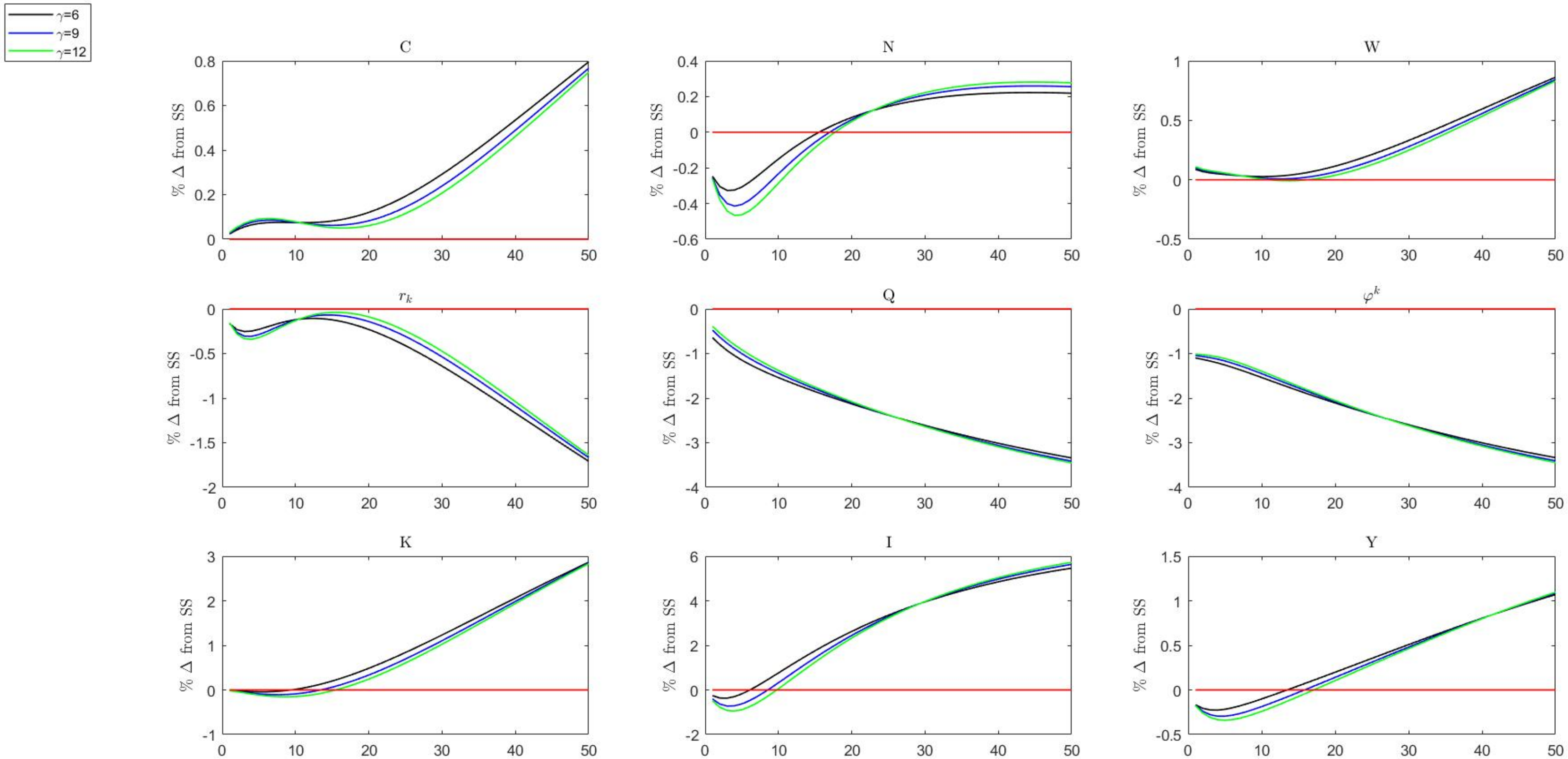

Figure 10: Impulse response functions to a permanent IST shock for different values of γ.

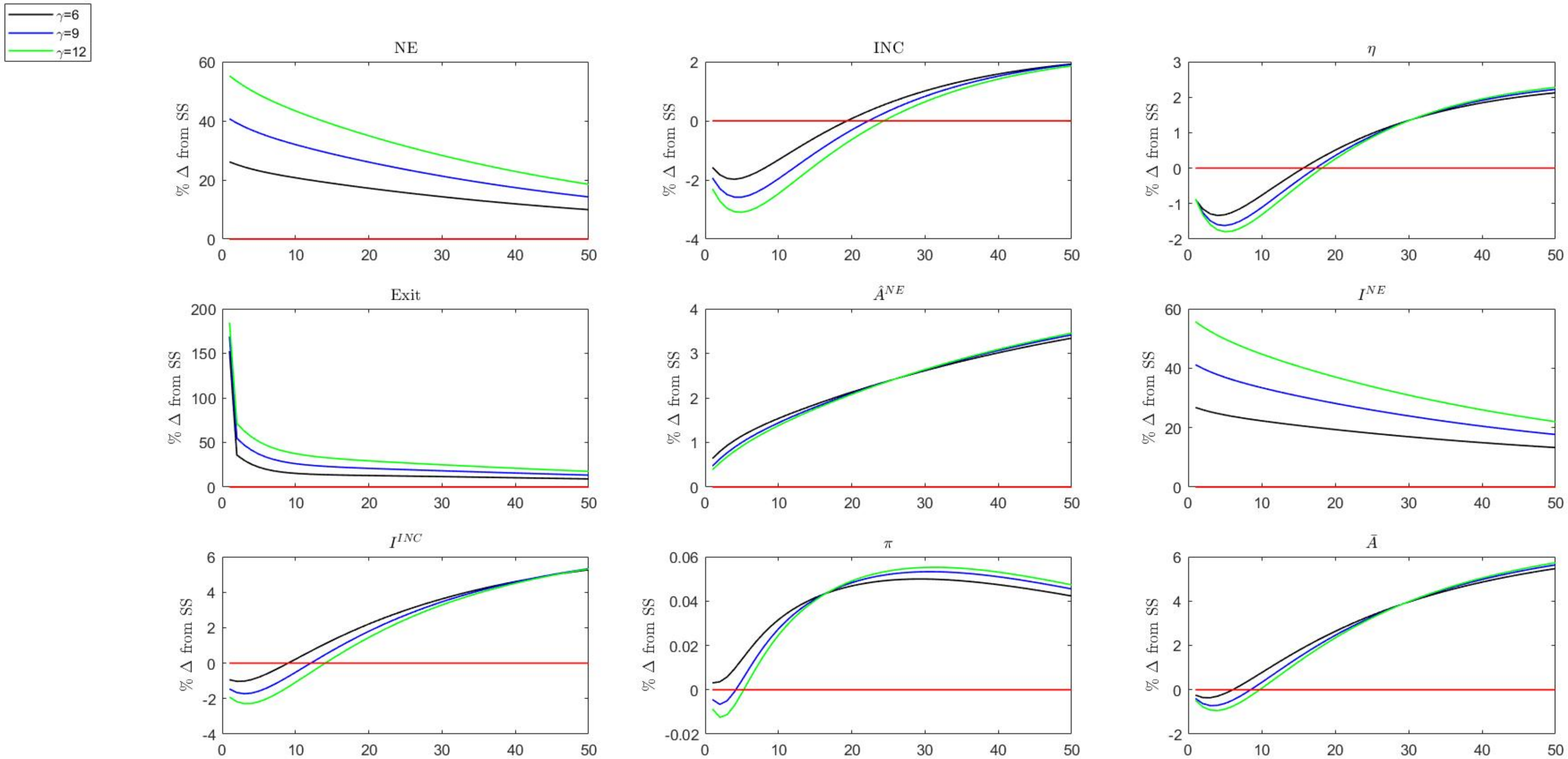

Figure 11: Impulse response functions to a permanent IST shock for different values of γ.

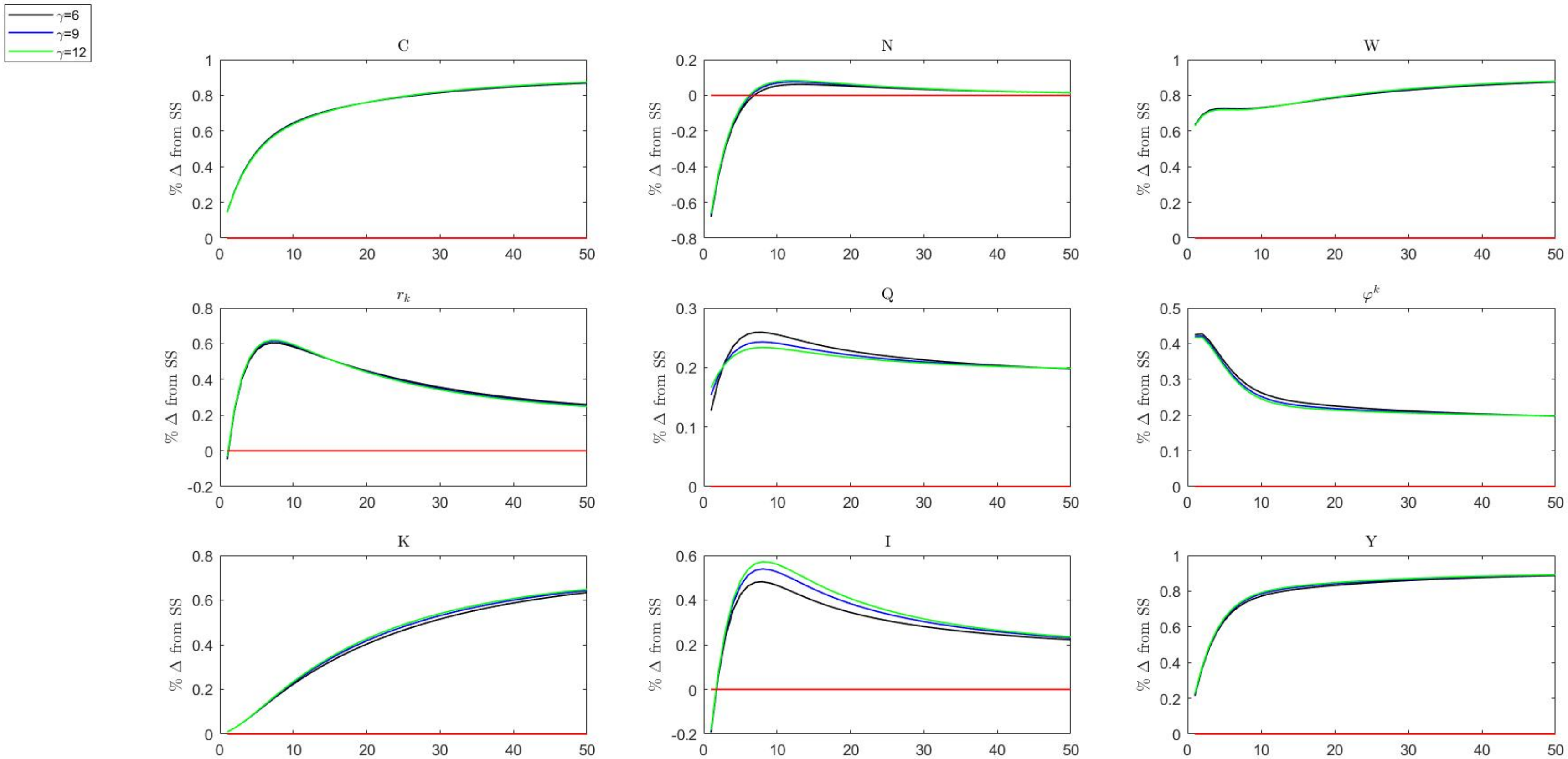

Figure 12: Impulse response functions to a permanent LAT shock for different values of γ.

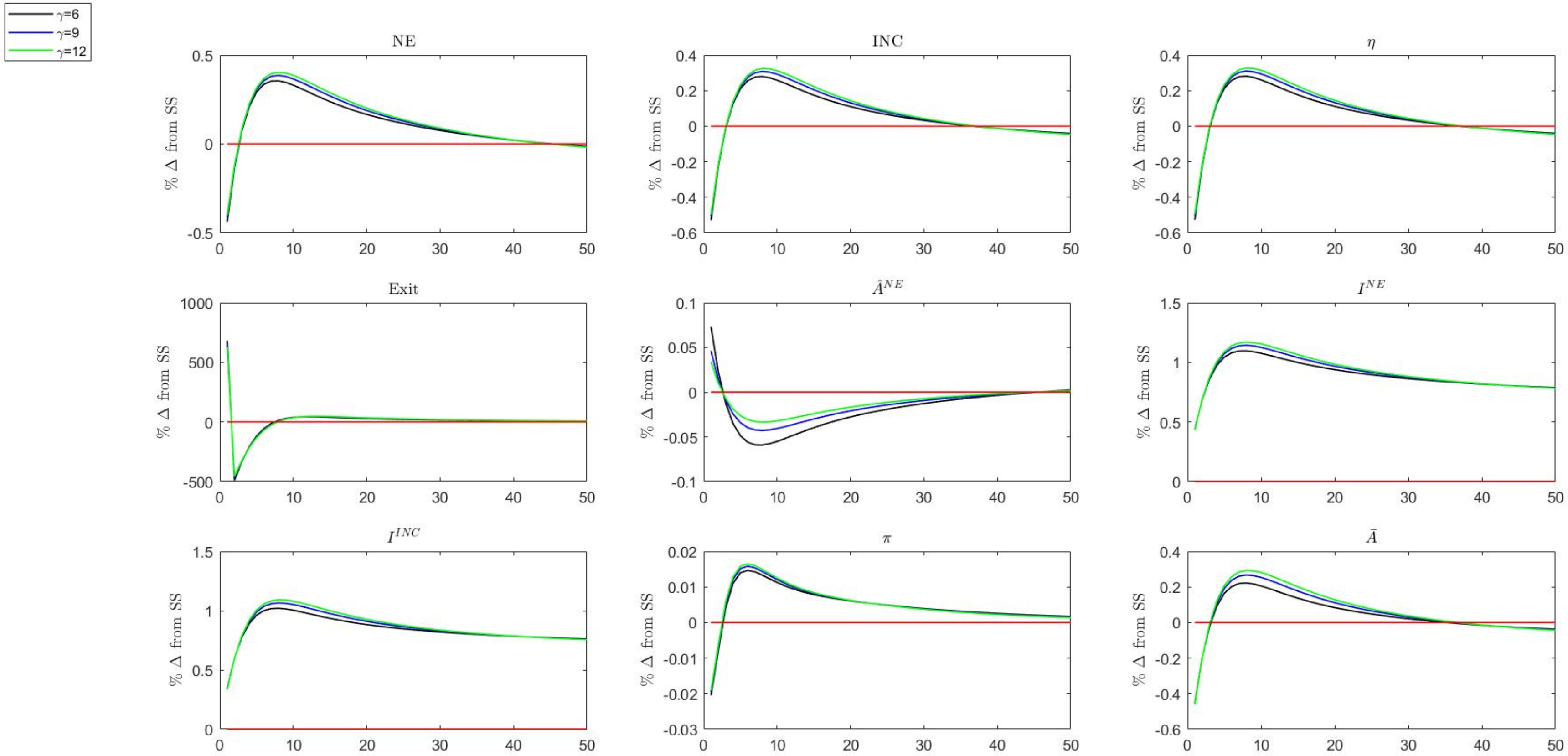

Figure 13: Impulse response functions to a permanent LAT shock for different values of γ.

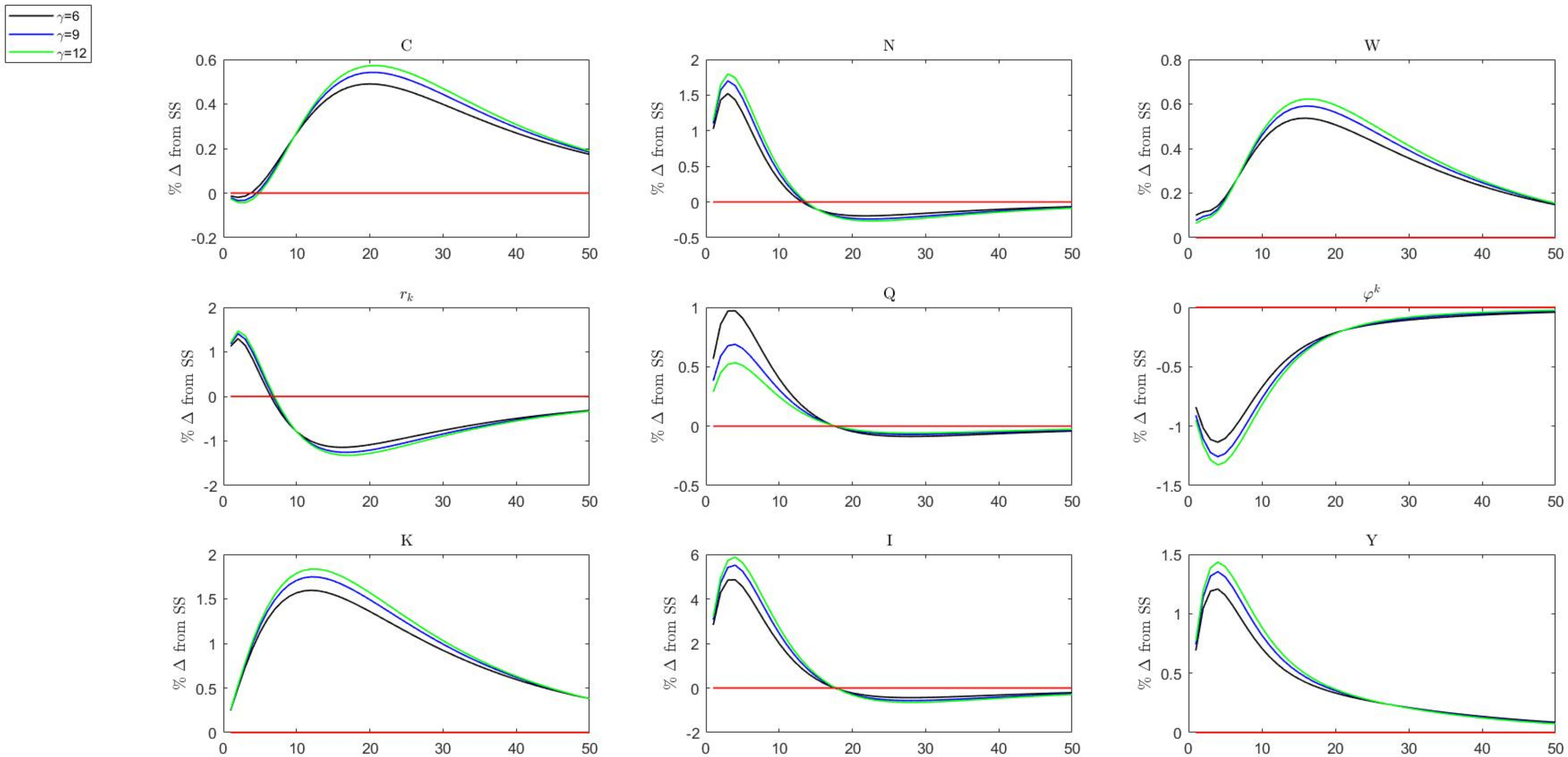

Figure 14: Impulse response functions to a transitory MEI shock for different values of γ.

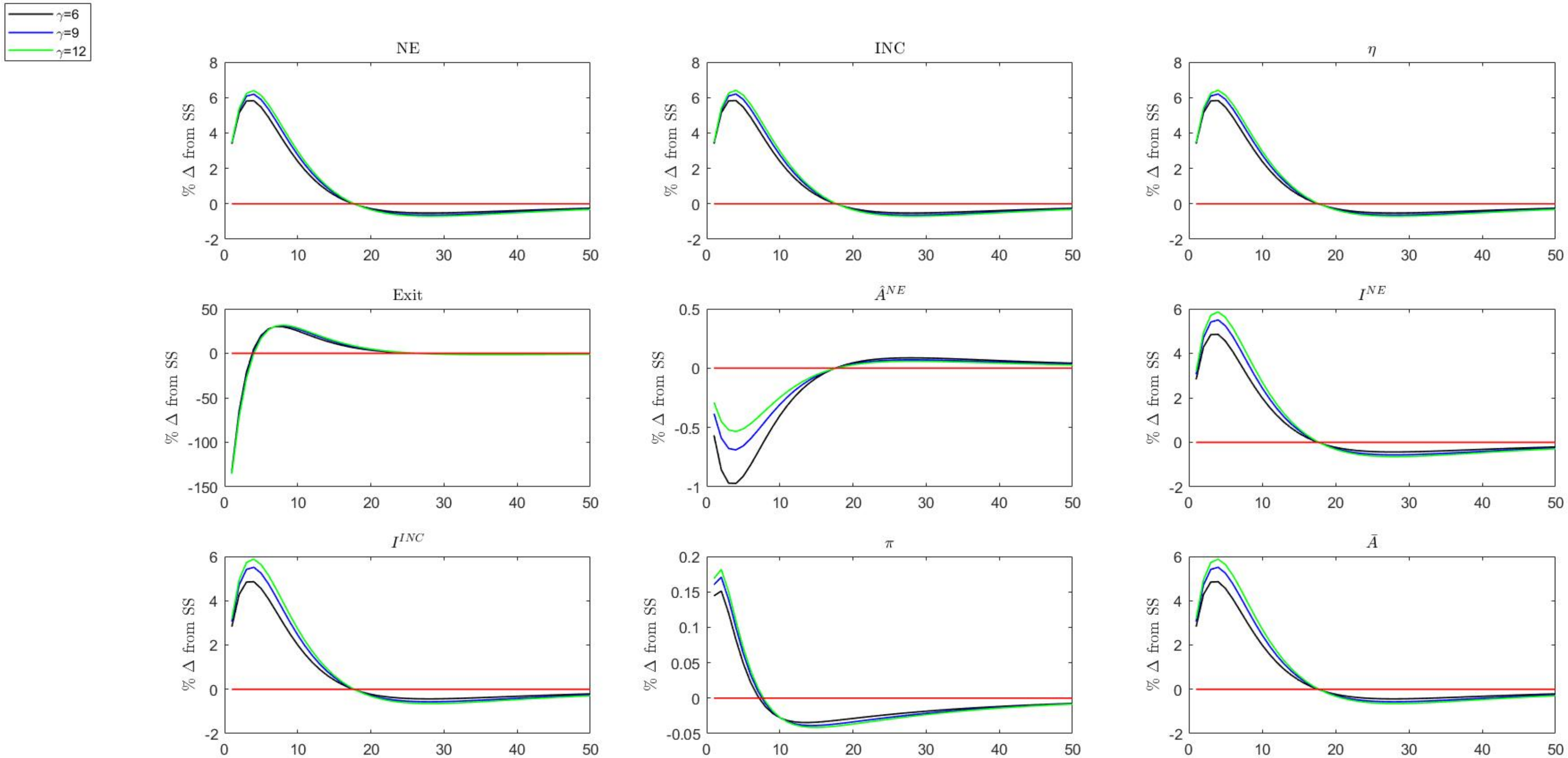

Figure 15: Impulse response functions to a transitory MEI shock for different values of γ.

G List of linearised equations

Here we list all the relevant linearised equations in our model. Detrended loglinearised variables are in small case and marked with a tilde.

G.1 Households

$$\widetilde{\lambda}_t^* = \frac{1}{(g_* - \beta a)(g_* - a)}\Big[a g_* (\beta \widetilde{g}_{t+1} - \widetilde{g}_t) - \left(g_*^2 + \beta a^2\right)\widetilde{c}_t + \tag{127}$$
$$+ g_* a \left(\widetilde{c}_{t-1} + \beta \widetilde{c}_{t+1}\right)\Big] \qquad \text{MUC}$$

$$\widetilde{w}_t = \theta \widetilde{N}_t - \widetilde{\lambda}_t^* \qquad \text{Labor Supply} \tag{128}$$

$$\widetilde{\lambda}_t^* = \widetilde{\lambda}_{t+1}^* - \bar{g}_{t+1} + $$
$$+ \frac{\beta}{g_*}\left[\frac{r_k^{ss}}{\widetilde{\varphi}^{k,ss}}\left(\widetilde{r}_{k,t+1}^* - \widetilde{\varphi}_t^{*,k}\right) + (1-\delta)\left(\widetilde{\varphi}_{t+1}^{*,k} - \widetilde{\varphi}_t^{*,k}\right)\right] \qquad \text{Capital Euler} \tag{129}$$

$$\widetilde{\lambda}_t^* = \widetilde{\lambda}_{t+1}^* - \widetilde{g}_{t+1} + \widetilde{r}_t \qquad \text{Bond Euler} \tag{130}$$

$$\widetilde{q}_t = \widetilde{\varphi}_t^{*,k} + \widetilde{\mu}_t^i + $$
$$+ \gamma_I g_*^2 \left[-\frac{(1+\beta g_*)}{g_*}\widetilde{i}_t + \widetilde{i}_{t-1} + \beta\left(\widetilde{i}_{t+1} + \bar{g}_{t+1}\right) - \bar{g}_t\right] \qquad \text{Investment rule} \tag{131}$$

G.2 Intermediate Producers

$$\widetilde{k}_t = \frac{(1-\delta)}{g_*}\left(\widetilde{k}_{t-1} - \bar{g}_t\right) + \frac{I^{ss}}{K^{ss}}(\widetilde{i}_t + \widetilde{\mu}_t^i) \qquad \text{Law of motion of capital} \tag{132}$$

$$\widetilde{y}_t = \chi \widetilde{N}_t + (1-\chi)\left(\widetilde{k}_{t-1} - \bar{g}_t\right) - \widetilde{\xi}_t \qquad \text{Final Output} \tag{133}$$

$$\widetilde{r}_{k,t}^* = \widetilde{mc}_t - \widetilde{\xi}_t + \chi\left(\bar{g}_t + \widetilde{N}_t - \widetilde{k}_{t-1}\right) \qquad \text{Demand of Capital} \tag{134}$$

$$\widetilde{w}_t = \widetilde{mc}_t - \widetilde{\xi}_t + (1-\chi)\left(\widetilde{k}_{t-1} - \widetilde{n}_t - \bar{g}_t\right) \qquad \text{Demand of Labor} \tag{135}$$

G.3 Final Producers

$$\widetilde{d}_t = \widetilde{\pi}_t^* + (1-\lambda_p\beta)\,\widetilde{y}_t + \\ + \lambda_p\beta\left[\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t - \widetilde{\pi}_{t+1}^* + (1-\nu)\left(\gamma_p\widetilde{\pi}_t - \widetilde{\pi}_{t+1}\right) + \widetilde{d}_{t+1}\right] \quad \text{First Recursive Inflation Term} \tag{136}$$

$$\widetilde{f}_t = (1-\lambda_p\beta)\left(\widetilde{y}_t + \widetilde{mc}_t\right) + \\ + \lambda_p\beta\left[\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t - \nu\left(\gamma_p\widetilde{\pi}_t - \widetilde{\pi}_{t+1}\right) + \widetilde{f}_{t+1}\right] \quad \text{Second Recursive Inflation Term} \tag{137}$$

$$\widetilde{d}_t = \widetilde{f}_t \quad \text{Inflation dynamics} \tag{138}$$

$$\widetilde{\pi}_t^* = \frac{\lambda_p}{1-\lambda_p}\left(\widetilde{\pi}_t - \gamma_p\widetilde{\pi}_{t-1}\right) \quad \text{Evolution of prices} \tag{139}$$

G.4 Capital Producers

$$\widetilde{\hat{a}}_t^{NE} = -\widetilde{q} \quad \text{NEs cutoff} \tag{140}$$

$$\widetilde{\hat{a}}_t^{INC} = -\widetilde{q} \quad \text{INCs cutoff} \tag{141}$$

$$\widetilde{\eta}_t^* = (1-H^{ss})\widetilde{ne}_t + H^{ss}\widetilde{inc}_t \quad \text{Mass of active K-producers} \tag{142}$$

$$\widetilde{ne}_t = -\gamma\widetilde{a}_t^{NE} \quad \text{NEs mass} \tag{143}$$

$$\widetilde{inc}_t = \widetilde{\eta}_{t-1}^* + \\ + \gamma\left[\widetilde{\hat{a}}_{t-1}^{NE} - \widetilde{\hat{a}}_t^{INC} + (\chi+\alpha-1)g_{z,t} - \chi\bar{g}_t\right] \quad \text{INCs mass} \tag{144}$$

$$\widetilde{exit}_t = \widetilde{\eta}_{t-1}^* + \\ - \frac{H^{ss}}{1-H^{ss}}\gamma\left[\widetilde{a}_{t-1}^{NE} - \widetilde{a}_t^{INC} + (\chi+\alpha-1)g_{z,t} - \chi\bar{g}_t\right] \quad \text{Exit Mass} \tag{145}$$

$$\widetilde{i}_t^{NE} = \widetilde{ne}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{NE} + \frac{\alpha}{1-\alpha}\widetilde{q}_t \qquad \text{NEs gross investment} \tag{146}$$

$$\widetilde{i}_t^{INC} = \widetilde{inc}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{INC} + \frac{\alpha}{1-\alpha}\widetilde{q}_t \qquad \text{INCs gross investment} \tag{147}$$

$$\widetilde{i}_t = \frac{I^{NE,ss}}{I^{ss}}\widetilde{i}_t^{NE} + \frac{I^{INC,ss}}{I^{ss}}\widetilde{i}_t^{INC} \qquad \text{K-producers gross investment} \tag{148}$$

$$\widetilde{s}_t = \frac{S^{NE,ss}}{S^{ss}}\left(\widetilde{ne}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{NE} + \frac{1}{1-\alpha}\widetilde{q}_t\right) + $$
$$+ \frac{S^{INC,ss}}{S^{ss}}\left(\widetilde{inc}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{INC} + \frac{1}{1-\alpha}\widetilde{q}_t\right) \qquad \text{K firms total input amount} \tag{149}$$

$$\widetilde{a}_t = \frac{NE^{ss}\hat{A}^{NE,ss}}{\bar{A}^{ss}}\left(\widetilde{ne}_t + \widetilde{a}_t^{NE}\right) + \frac{INC^{ss}\hat{A}^{INC,ss}}{\bar{A}^{ss}}\left(\widetilde{inc}_t + \widetilde{a}_t^{inc}\right) \qquad \text{K-sector productivity} \tag{150}$$

G.5 Market clearing conditions and policy rules

$$\widetilde{y}_t = \frac{C^{ss}}{Y^{ss}}\widetilde{c}_t + \frac{S^{ss}}{Y^{ss}}\widetilde{s}_t + \frac{NE^{ss}f^{NE}}{Y^{ss}}\widetilde{ne}_t + \frac{INC^{ss}f^{INC}}{Y^{ss}}\widetilde{inc}_t \qquad \text{Market clearing} \tag{151}$$

$$\widetilde{r}_{b,t} = \rho_{R_b^{ss}}\widetilde{r}_{b,t-1} + (1-\rho_{R_b^{ss}})\left(k_\pi\widetilde{\pi}_t + k_y\widetilde{mc} + \sigma^r\varepsilon_{rt}\right) \qquad \text{Taylor rule} \tag{152}$$

$$\widetilde{r}_{b,t} = \widetilde{r}_t + \widetilde{\pi}_{t+1} \qquad \text{Fisher equation} \tag{153}$$